Abstract

We study exit decisions of duopolists from a stochastically declining market. Over time, firms privately learn about market conditions from observing the stochastic arrival of customers. Exit decisions are publicly observed; thus the model features both observational and private learning. A larger firm is more likely to have customers and hence has better information about market conditions than does a smaller rival. We provide sufficient conditions for either the smaller or the larger firm to be the first to exit the market in the unique equilibrium. Uniqueness follows from iterated conditional dominance: because of observational learning, exiting may be a firm’s dominant action since continuing operation would bring too much of a good news to the rival, leading it to further postpone its exit.

Keywords: Duopoly, Exit, Private Learning, War of Attrition

JEL Codes: C73, D21, D43, D82, D83

1 Introduction

The empirical evidence on exit from a declining industry has shown that the relationship between firm size and exit patterns varies across different industries and may depend on industry-specific characteristics. Some studies have found higher rates
of closure for small firms (see, for example, Lieberman, 1990). Other studies have
documented that in mature stages of the industry life cycle, and particularly in tech-
nically advanced industries, smaller-scale firms are not necessarily confronted with
a lower likelihood of survival relative to their larger counterparts (see Agarwal and
Audretsch, 2001). As pointed out by Besanko et al. (2010), the existing theoretical
literature fails to explain why the producer that adjusts to fluctuations in demand
can be either the large firm or the small firm.

The theoretical literature has modeled strategic exit from a declining industry
using the war of attrition paradigm, predicting that a stronger firm can force a weaker
firm to exit first. Starting with the seminal contributions of Ghemawat and Nalebuff
(1985), Fudenberg and Tirole (1986), and Fine and Li (1989) (see also Murto, 2004),
the majority of papers have identified a firm’s strength with its profit flow: a firm is
stronger than its competitor if its profit flow is greater. To the best of our knowledge,
no existing model explains why sometimes the firm that survives the industry decline
is the one with the lowest profit flow.

Our paper offers a novel explanation for the seemingly unsettled correlation be-
tween profits and likelihood of survival in a declining industry. When a firm pri-
vately learns about the profitability of the industry, for example, from sales data,
its strategy—whether it exits or not—conveys information to its competitor. As a
result, a firm’s relative strength is determined not only by its profit flow but also by
the information externalities generated by its actions.

To model dynamic selection in a declining industry, we consider an irreversible
timing game. Initially, each duopolist earns positive expected profits; the industry
randomly transitions to a declining stage of its life cycle, unbeknown to either firms. In
this declining stage, the duopoly loses viability as the expected profit of each duopolist
becomes negative. Firms privately learn about the profitability of the industry by
observing their customer arrivals. We focus on the case in which firms are asymmetric
in that they have different customer arrival rates and hence learn at different speeds.\footnote{We believe the mechanism at play is best illustrated in this set up because whenever firms are sufficiently asymmetric, there exists a unique equilibrium. However, the equilibria we construct are also equilibria of the symmetric game.} Since customer arrivals are privately observed and exit decisions are public, the model
features both private and observational learning, akin to an incomplete information
war of attrition.
First, we show that there always exists an equilibrium in which one of the two firms exits first with probability one. To determine which firm survives, we consider each firm’s best reply to the other never exiting. In our model, it is the first exit time when playing this best reply\(^2\) that determines a firm’s strength. The smaller the first exit time is, the weaker the firm. There always exists an equilibrium in which the weaker firm exits first with probability one.

Intuitively, the stronger firm has incentives to wait for the news revealed at the weaker firm’s first exit time. Furthermore, if the weaker firm does not exit at that time, the stronger firm’s incentive to remain in the market is reinforced. The weaker firm, by staying in the market, sends a signal that is against its own interest and that makes the stronger firm more optimistic about the market conditions—yet, in equilibrium, it cannot avoid doing so. As a result, the weaker firm is discouraged from remaining in the market longer compared to the case in which it expects never to enjoy monopoly profits nor to benefit from observing the other firm’s action.

We show that there is a non-monotone relationship between the speed of private learning (or, equivalently, the firm’s expected profit flow) and the firm’s strength in the war of attrition (or, equivalently, its first exit time). The non-monotone relationship arises because of two countervailing forces. On the one hand, the higher the customer arrival rate is, the faster the firm becomes pessimistic about the market conditions if no customer shows up. On the other hand, the higher the customer arrival rate is, the higher the expected profit flow, and the stronger the incentive to remain in the market for any belief regarding the state of the industry.

Second, we provide sufficient conditions for the equilibrium in which the weaker firm exits first to be the unique equilibrium of the game. In light of the non-monotonicity between a firm’s customer arrival rate and its strength, in the unique equilibrium either the larger or the smaller firm survives. As both predictions have received some empirical support, our model sheds light on how industry characteristics can affect the equilibrium outcome. Roughly, if there is a high degree of uncertainty, i.e., both firms have little information about the market conditions, then the smaller firm exits first with probability one; if the larger firm has (sufficiently) precise information, it exits first with probability one.

\(^2\)That is, the earliest time the firm exits with positive probability along the path induced by the best reply.
Our model relies on the canonical exponential bandit framework with inconclusive good news. We start by proving equilibrium uniqueness for the special case of conclusive news, that is, the case where no customer arrives in the declining phase of the industry. Then, we show that the result generalizes to the case of inconclusive news.

The proof relies on the iterated deletion of (conditionally) dominated strategies à la Shimoji and Watson (1998). In principle, to identify dominated strategies, one needs to compute the beliefs of a firm for any given strategy of the rival. This computation proves to be difficult in our setup because higher-order beliefs play a key role, not only because a firm needs to forecast its opponent’s action but also because firms’ private signals are correlated. Standard techniques do not apply: since the underlying state of the world evolves over time, it is not possible to simplify the dynamic inference problem by decomposing the posterior belief into two single-dimensional statistics, i.e., a private belief and a public belief, such as in Foster and Viswanathan (1996) and Rosenberg, Solan, and Vieille (2007), as discussed in Section 4.2.

We circumvent these difficulties by providing a recursive lower bound to a firm’s posterior belief about the prevailing state in any equilibrium of the game, an approach that could be applied to other models with private learning or private monitoring. In a first step, we compute a lower bound to the stronger firm’s posterior belief conditional on the weaker firm not using a strictly dominated strategy. In a second step, we use this lower bound to identify an initial interval of time when continuing operations is a conditionally dominant strategy for the stronger firm, irrespective of its private history. In a third step, we show that exiting is initially dominant for the weaker firm whenever the time elapsed since last observing a customer is sufficiently long. We also show that, in the special case of conclusive news, the stronger firm’s inference problem exhibits a recursive structure, which allows us to generalize the dominance argument to countably many rounds of deletion, concluding that the stronger firm never exits in equilibrium.

Introducing inconclusive news disrupts the recursive properties of the inference problem. Our proof relies instead on an approximation argument. In the limit as the weaker firm’s information becomes arbitrarily precise, along any history, the stronger firm’s belief in the game with conclusive news provides a good approximation of its belief in the game with inconclusive news, allowing us to conclude that there
exists a unique equilibrium. Moreover, we show that introducing asymmetry in other dimensions such as discount rate, cost or revenue does not change our main results.

1.1 Literature Review

Our work is closely related to the theoretical literature analyzing exit through the lens of the war-of-attrition paradigm. Ghemawat and Nalebuff (1985, 1990) study disinvestment in declining industries when the demand shrinks deterministically over time. Applying a backward induction argument, Ghemawat and Nalebuff (1985) show that in the unique equilibrium the larger firm exits first because it is unable to adjust capacity and loses viability more quickly. Murto (2004) shows that the main insights of Ghemawat and Nalebuff (1985) carry over to the case of stochastic market decline and a general payoff structure: there always exists an equilibrium in which the firm with the lowest expected profit flow exits first. However, this equilibrium is not unique if the degree of uncertainty is high. In contrast to our model, in Murto (2004), signals about the underlying uncertainty, which is modeled as a geometric Brownian motion, are public, precluding signaling effects.

Incomplete information in a war of attrition has been studied in Fudenberg and Tirole (1986), who characterize the unique equilibrium of the exit game when firms have private information about their outside option or their cost parameter. In contrast to Fudenberg and Tirole (1986), information is interdependent in our model; thus, higher-order beliefs are relevant not only to form beliefs about the strategy of the opponent but also to assess the prevailing state of the world. Takahashi (2015) empirically estimates the model of Fudenberg and Tirole (1986) using the US movie theater industry to quantify the welfare loss from strategic delay.

Within the broader literature on stopping games, this paper is closely related to Rosenberg, Solan, and Vieille (2007) and Murto and Välimäki (2011). While these papers are concerned with different questions, they feature, similar to ours, observational learning and irreversible action, but they do not incorporate payoff externalities. In contrast, Hopenhayn and Squintani (2011) and Gorno and Iachan (2020) feature payoff externalities and independent private information.4

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3 See also the discrete time model of Fine and Li (1989).
4 While these papers focus on symmetric setups and equilibria, in our equilibrium, as in the asymmetric equilibria of the model of Kirpalani and Madsen (2019), only one firm reaps the benefit of observational learning.
More recently, Awaya and Krishna (2020) show that information can be a strategic disadvantage in an R&D race with private information: under some parametric restrictions, in the unique equilibrium, the better-informed firm loses the race more frequently and has higher payoffs.\footnote{Chen and Ishida (2020) study an asymmetric war of attrition with independent types and dynamic private learning. Similarly to Awaya and Krishna (2020), in some equilibrium, the less-efficient firm wins more often. See also Kim and Lee (2014) who study the affect of information acquisition in war of attrition.} In our view, both in Awaya and Krishna (2020) and in our model, the signaling consequences of a firm’s action affect the option value of waiting. Our results show that with dynamic private learning, observation learning can operate in favor or against the better-informed firm. Specifically, while in Awaya and Krishna (2020), better information always becomes a competitive disadvantage, in our model, there is a non-monotone relationship between strength and the speed of private learning.

Our model is also tangentially related to the literature on strategic experimentation with exponential bandits. As in Keller and Rady (2010), firms learn via inconclusive good news, but as in Keller and Rady (1999, 2003), the underlying state of the world changes over time.\footnote{See also Khromenkova (2018).} Rosenberg et al. (2013) and Heidhues et al. (2015), who analyze experimentation models with private learning, are also related.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 analyzes two public learning benchmarks: the case of observable decline and the case of observable customers. Section 4 is devoted to the main results. Section 5 shows that the model is equivalent to an irreversible investment game with a first mover advantage, and Section 6 concludes. The Appendix contains all the omitted proofs.

## 2 Model

Time is continuous, and the horizon is infinite, $t \in [0, \infty)$. Two firms decide when to irreversibly exit a declining industry. Each firm’s present discounted payoff from exiting the industry is normalized to 0.

The industry profitability is determined by a state of the world $\omega_t$ that can be either good or bad, $\omega_t \in \{G, B\}$. Initially, both firms attach probability one to the industry being profitable, $\omega_0 = G$. The industry irreversibly becomes unprofitable,
unbeknown to the two firms, at some random time which is exponentially distributed with parameter $\gamma > 0$.\(^7\)

Each active firm serves a stream of randomly arriving customers. In a duopoly, that is, as long as both firms are active, the customers of firm $i$ arrive according to an inhomogeneous Poisson process with intensity $\lambda^i_{\omega t}$, where $\lambda^i_G > \lambda^i_B \geq 0$. In a monopoly, that is, after firm $j$ exits, the customers of firm $i$ arrive at a rate $\lambda^i_{\omega t} + \lambda^j_{\omega t}$. Each firm privately observes its customer arrival while exit decisions are public.

While active, each firm bears a flow cost $c$, and each customer yields a lump-sum revenue $R$. Firms discount the future at a common rate $r > 0$. We impose the following parametric assumptions.\(^8\)

**Assumption 1.** For $i = 1, 2$,

$$\lambda^i_B R - c < 0.$$ \(^9\)

Assumption 1 states that a duopolist’s flow payoff is negative whenever $\omega_t = B$: it ensures that it can ever be optimal to exit. Furthermore, we assume that $\lambda^2_B = \lambda^1_B = \lambda^B$.\(^9\)

After one of the firms has exited, the remaining firm enjoys monopoly profits until it finds it optimal to exit as well. A strategy for firm $i$ dictates when to exit along any nonterminal history as a function of all the information available. Formally, a strategy of firm $i$, $\sigma^i$, is a stopping time adapted to the filtration generated by the inhomogeneous Poisson process of customer arrivals, $N^i_t$, and the exit decision of player $j$, i.e., whether and when player $j$ has exited.

Given a strategy profile $(\sigma^1, \sigma^2)$, the payoff of firm $i$ can be written as:

$$\mathbb{E}^{(\sigma^1, \sigma^2)} \left[ \int_0^{\sigma^i} e^{-rt} \left( \lambda^i_{\omega t} R + 1_{\{\sigma^j < t\}} \lambda^j_{\omega t} R - c \right) dt \right]. \quad (1)$$

\(^7\)Our model generalizes to the case of $\gamma = 0$ and interior prior about $\omega_0$. The assumption that the distribution of the time when the state transitions is exponential is convenient but not essential. The results can easily be generalize, for example, to any distribution with a bounded hazard rate.

\(^8\)We could allow the flow cost to depend on the presence of a competitor, as long as the expected profit conditional on the state is strictly higher in a monopoly than that in a duopoly. For a discussion on asymmetries, see Section 4.4.

\(^9\)Our model can accommodate both the case in which a monopolist’s profit is always profitable and the case in which a monopolist’s profit is negative whenever $\omega_t = B$. 

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We focus on Perfect Bayesian equilibria of the stochastic timing game. However, we do not explicitly specify beliefs and behavior off the equilibrium path because they play no role in sustaining on-path behavior, given that the payoff from exiting is independent of the behavior of the remaining firm and exit is irreversible.\textsuperscript{10}

As in canonical bandit models (e.g., Rothschild, 1974), customers bring revenue and information. Our results rely on the more general assumption of a positive correlation between the expected profit of a firm and the precision of its information. For example, one could argue that a larger firm enjoys higher profits because of higher markups and economies of scale\textsuperscript{11} and that it has more precise information as a result of a more sophisticated market research department.

\section{Benchmarks}

To put our results into perspective, we start by discussing two benchmarks. In the first benchmark, the state transition is observable. In the second, firms do not observe the state transition, but, as in Murto (2004), they observe each others’ customers.

\subsection{Observable Decline}

As long as the state is good, i.e., $\omega_t = G$, it is dominant for both firms to remain in the market. If $2\lambda BR - c > 0$, the continuation game after the state transitions is a standard war of attrition, as in (Hendricks et al., 1988), which is known to have a multiplicity of equilibria.

Specifically, it has two pure strategy asymmetric equilibria. In each of them, one firm exits as soon as the state transitions while the other firm never exits. There exists a mixed strategy symmetric equilibrium in which both firms exit at the constant rate

$$\frac{-r}{2\lambda BR - c}$$

such that the cost of waiting, $c - \lambda BR$, equals the benefit, $\varsigma (2\lambda BR - c) / r$, where $\varsigma$ is the equilibrium exit rate of each firm.

\textsuperscript{10}In our game, any Nash equilibrium is outcome-equivalent to a Perfect Bayesian equilibrium.

\textsuperscript{11}The empirical literature finds higher markups for larger firms, e.g., De Loecker and Warzynski (2012), Autor et al. (2020), Edmond et al. (2018) and Boar and Midrigan (2019).
Hence, a model with an observable state is silent about how industry characteristics affect the likelihood that the smaller or the larger firm survives the industry decline. In contrast, with private learning, the equilibrium is unique, and we can identify the conditions under which either the smaller or the larger firm exits first.

3.2 Public Learning

If firms observe each other’s customers, the model is closely related to the model of Murto (2004). Despite the difference in the stochastic process governing the underlying state of the world, the main insights of Murto (2004) carry over to our setup, as formalized by the following proposition.

**Proposition 1.** If both firms observe each other’s customers and $\lambda_2^G > \lambda_1^G > c/R$,\(^{12}\) there exists an equilibrium in which firm 2 never exits first.

As in Murto (2004), whenever the game has a unique Markov equilibrium, the larger firm forces the weaker one to exit first. In other words, a firm’s “strength” is monotone in its profit flow:\(^{13}\) the larger $\lambda_i^G$ is, the longer the firm is willing to remain in the market. When instead there exist multiple equilibria and, in particular, there exists an equilibrium in which the smaller firm never exits first, there also exists a mixed strategy equilibrium.

As we discuss in the Appendix, in the mixed strategy equilibrium, the firm with the higher customer arrival rate exits with positive probability at a certain belief; for lower beliefs, both firms exit at a positive rate.\(^{14}\) In equilibrium, the rate at which a firm exits makes the opponent indifferent between exiting and remaining in the market. Consequently, as in a nondegenerate equilibrium of a complete information war of attrition, the firm with the larger customer arrival rate has a lower probability of survival.

It has been argued that it is odd that in the mixed strategy equilibrium, the firm with the smaller customer arrival rate (i.e., the largest cost of fighting) wins more

\(^{12}\)We discuss the bound $\lambda_1^G > c/R$ in Lemma 1 below.

\(^{13}\)In Murto (2004) a firm’s “strength” is determined by its profit flow when it becomes a monopolist. In our model, the payoff and the behavior of a firm once it becomes a monopolist are irrelevant as far as our equilibrium construction is concerned.

\(^{14}\)The existence of a mixed-strategy equilibrium, seemingly in contrast to Georgiadis et al. (2020), relies on the stochastic properties of the belief process; in Georgiadis et al. (2020) the belief is a Brownian motion.
frequently. Hence, when multiple equilibria exist, the most realistic equilibrium seems to be the one in which the firm with the larger customer arrival rate never exits first. (See, for example, Kornhauser et al., 1989.) In a sense, our model provides a rationale for the larger firm to concede more often in equilibrium: interdependent values and observational learning.

4 The Private Learning Game

This section discusses our main results. We first construct two candidate equilibria. We then provide sufficient conditions for each of them to be the unique strategy profile that survives iterated deletion of (conditionally) dominated strategies.

4.1 A Best-Reply Problem

Suppose firm $j$ adopts the strategy of never exiting the market, and consider the best-reply problem of firm $i \neq j$. The problem of firm $i$ can be written as a standard optimal stopping problem:

$$\sup_{\tau} E \left[ \int_0^{\tau} e^{-rt} (\lambda_i^i \omega_t R - c) \, dt \right].$$

The problem of firm $i$ is Markov in its posterior belief about the prevailing state and the best response takes a simple form: it prescribes exiting as soon as the posterior belief falls below some cutoff $\pi^*(\lambda_i^G)$. Define $\tau^*(\lambda_i^G)$ as the earliest time firm $i$ exits with positive probability along the path induced by the best-reply strategy, that is,

$$\Pr \left[ \omega_{\tau^*(\lambda_i^G)} = G \mid N_{\tau^*(\lambda_i^G)}^i = 0 \right] = \pi^*(\lambda_i^G).$$

In other words, along the history with no customers, firm $i$ exits at time $\tau^*(\lambda_i^G)$. (Recall that $N_i^i$ denotes the inhomogeneous Poisson process of customer arrivals of firm $i$.)

In the special case of conclusive news, i.e., $\lambda_B = 0$, $\tau^*(\lambda_i^G)$ fully characterizes the best reply. Because the posterior belief about the prevailing state jumps to one whenever the firm observes a customer, the best reply prescribes exiting as soon as no customers have been observed for an uninterrupted amount of time of length $\tau^*(\lambda_i^G)$.

The following lemma characterizes how $\pi^*(\lambda_i^G)$ and $\tau^*(\lambda_i^G)$ change with $\lambda_i^G$. 

Figure 1: Best reply for \((c, R, r, \gamma) = (1/2, 1, 1/10, 1/5)\). The solid line indicates the case of conclusive news, \(\lambda_B = 0\). The dashed line indicates a case of inconclusive news, \(\lambda_B = 1/5\).

**Lemma 1.**

(i) \(\pi^*(\lambda^i_G) < 1\) if and only if \(\lambda^i_G R > c\).

(ii) \(\tau^*(\lambda^i_G)\) is single-peaked and \(\lim_{\lambda^i_G \to \infty} \tau^*(\lambda^i_G) = 0\).

Intuitively, if \(\lambda^i_G\) is sufficiently low, it is too unlikely that the firm will be able to serve enough customers to make it worthwhile to remain in business. In light of (i), in the remainder of the paper, we assume that \(\lambda^i_G R > c\), \(i = 1, 2\).

The non-monotonicity, illustrated in the right panel of Figure 1, is due to two countervailing forces. On the one hand, the higher \(\lambda^i_G\) is, the higher the marginal benefit from remaining in the market at any given belief. In fact, the cutoff belief \(\pi^*(\lambda^i_G)\) is decreasing in \(\lambda^i_G\), as in the left panel of Figure 1. On the other hand, the higher \(\lambda^i_G\) is, the faster the firm becomes pessimistic about the market conditions.\(^{15}\)

This observation is at the core of our main results.

In contrast to the case of privately observed customers, when learning is public, as in Section 3.2, a firm’s first exit time is monotone in the rate of arrival of customers,\(^{15}\)

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\(^{15}\)The result is reminiscent of Halac, Kartik, and Liu (2016) and Bobtcheff and Levy (2017); our result generalizes theirs to a setup with inconclusive news and changing state.
that is, it is monotone in its profit flow. In fact, the single-peakedness in Lemma 1 relies on the fact that as $\lambda_G$ increases, both the profit flow and the speed of learning increase.\[^{16}\] With observable customers, the second force plays no role and there always exists an equilibrium in which the firm with the higher customer flow never exits first. This non-monotonicity is the main reason why we deliver different predictions from those of existing models.

### 4.2 A pure strategy equilibrium

Since $\pi^*(\lambda_G)$ is the optimal exit cutoff under the most pessimistic scenario in which the other firm never exits first, in equilibrium, exiting when the posterior belief is larger than $\pi^*(\lambda_G)$ is a dominated strategy, as formalized below.

**Lemma 2.** In any equilibrium, if firm $i$ exits with positive probability at time $t$ along some history in which firm $j$ is still active, then

\[
\Pr[\omega_t = G \mid (N_i^t)_{s \leq t}, \sigma^j \geq t] \leq \pi^*(\lambda_G)
\]

**Proof.** Consider firm $i$ at time $t$ following the private history $(N_i^t)_{s \leq t}$, and assume

\[
\Pr[\omega_t = G \mid (N_i^t)_{s \leq t}, \sigma^j \geq t] > \pi^*(\lambda_G)
\]

Hence, if the firm remains in the market until $t + dt$, the expected payoff it collects in $[t, t + dt)$ is bounded below by

\[
\left( -c + (\Pr[\omega_t = B \mid (N_i^t)_{s \leq t}, \sigma^j \geq t]\lambda_B + \Pr[\omega_t = G \mid (N_i^t)_{s \leq t}, \sigma^j \geq t]\lambda_G) \right) \left( R + v \left( \frac{\pi^*(\lambda_G)\lambda_G}{\pi^*(\lambda_G)\lambda_G + (1 - \pi^*(\lambda_G))\lambda_B} \right) \right) dt > 0,
\]

where $v : [0, 1] \rightarrow \mathbb{R}$ denotes the value function associated with the best-reply problem in Section 4.1. The bound follows from a few observations. First, the last term on the left-hand side is a lower bound to the expected continuation payoff after observing a

\[^{16}\]It can also be shown that with payoff-irrelevant signals, assuming that the (unobservable) flow payoff is positive if and only if the state is good, the first exit monotonically decreases in the learning speed. Details are available upon request.
customer: firm $i$ can only benefit from firm $j$ using a strategy other than never exiting. Second, in $[t, t + dt)$, firm $j$ may exit the market, but because the expected payoff in the continuation game is weakly positive, we can omit the corresponding term. Last, by definition of $\pi^*(\lambda^i_G)$, the inequality holds for any $\Pr[\omega_t = G \mid (N^i_{s})_{s \leq t}, \sigma^j \geq t] > \pi^*(\lambda^i_G)$. Therefore, the result follows.

In any equilibrium, a firm’s belief about the underlying state of the world evolves because of private and observational learning. In light of existing results, such as those of Rosenberg et al. (2007), one may expect to be able to decompose a player’s posterior belief into two single-dimensional statistics, i.e., a private belief and a public belief. Unfortunately, there exist no single-dimensional statistics that, combined with the private belief, yield the posterior belief about the prevailing state of the industry.

Intuitively, because the state is not perfectly persistent, the fundamental uncertainty concerns not only whether the industry has already become unprofitable but also when that happened. In fact, conditional on the prevailing state being bad, i.e., $\omega_t = B$, players’ private signals are correlated, making the standard decomposition technique inapplicable.

As a result, the relevant information for firm $i$ cannot be summarized by its private belief about the prevailing state and the status of the other firm (active or not). The second-order belief of firm $i$, that is, its distribution over the private belief firm $j$, affects firm’s $i$ posterior about the current state of the world in a more complicated manner than in Rosenberg et al. (2007).

Our first main result identifies an equilibrium in which firms’ inference problems are uncomplicated. A class of strategies that generate a simple inference problem are cutoff strategies. According to a cutoff strategy, for some measurable function $p_t : [0, \infty) \to [0, 1]$, a firm exits with probability one at the first-passage time of its posterior belief under $p_t$. For any $p > 0$, let $\sigma^i_p$ be the pure strategy according to which firm $i$ adopts a time-independent cutoff $p$. Let $\sigma^i_0$ be the strategy that prescribes never exiting.

**Theorem 1.** Fix the customer arrival rate of firm 1. If the customer arrival rate of firm 2 is high enough, there exists an equilibrium in which firm 2 (the larger firm) exits from the industry with probability one, i.e., $(\sigma^1_0, \sigma^2_\pi(\lambda^i_G))$ is an equilibrium.

\footnote{In a similar vein, Cisternas and Kolb (2020) show that in a private monitoring setup, the second-order beliefs can be decomposed in a similar vein. Yet, in their setup, in equilibrium, the first- and second-order beliefs are determined by a finite-dimensional sufficient statistic.}
Lemma 1. First, by definition, always exists an equilibrium in which either the smaller or the larger firm exits first \( \tau \) for the case of inconclusive news is relegated to the Appendix. We claim that if news, stronger: in the case of conclusive news, for any pair which firm 2 exits first with probability one. However, the proof shows something depending on whether \( \sigma \). To gain some insight into the learning dynamics, Figure 2 illustrates a possible realization of belief paths for the equilibrium in Theorem 1 under the assumption that \( \lambda^2_G > \lambda^1_G > \lambda_B = 0 \) and \( \tau^*(\lambda^2_G) < \tau^*(\lambda^1_G) \). In the interval \([0, \tau^*(\lambda^2_G))\), observational learning plays no role. No firm is supposed to exit, and both base their assessment of the market profitability on their private signals only. Notice that in the absence of arrivals, the belief of firm 2 decreases at a faster rate because \( \lambda^2_G > \lambda^1_G \).

In the illustrated equilibrium outcome, firm 2 does not exit at \( \tau^*(\lambda^2_G) \) because it observes a customer before that time. As shown in Figure 2, at \( \tau^*(\lambda^2_G) \), firm 1’s belief jumps upward as firm 2 not exiting reveals that it has observed a customer in \([0, \tau^*(\lambda^2_G))\). In the example, firm 1 does not observe any customer in \([0, \bar{t})\). As a result, at any \( t \in [\tau^*(\lambda^2_G), \bar{t}) \), the belief of firm 1 about the state of the world, as well as its second-order belief, is constant.\(^{19}\) as illustrated in Figure 3. Firm 2

\(^{18}\)We discuss this fact at length when explaining Figure 2 below.

\(^{19}\)The first- and second-order beliefs of firm 1 would not be constant in the case of inconclusive news, \( \lambda_B \neq 0 \).
Figure 2: Example of equilibrium belief trajectories for \((c, R, r, \gamma, \lambda_G^1, \lambda_G^2, \lambda_B) = (1/2, 1, 1/10, 1/5, 4, 1, 0)\). The solid line is the belief trajectory of firm 1. The dashed line is the belief trajectory of firm 2. The vertical line demarcates the time at which firm 2 exits.

Figure 3: The first- and second- order belief of firm 1 in the equilibrium \((\sigma_0^1, \sigma_2^*(\lambda_G^2))\) for \((c, R, r, \gamma, \lambda_G^1, \lambda_G^2, \lambda_B) = (1/2, 1, 1/10, 1/5, 4, 1, 0)\). On the left, the posterior about the prevailing state at some time \(t > \tau^*(\lambda_G^2)\) as a function of the time elapsed since last observing a customer. On the right, firm 1’s distribution over firm 2’s posterior belief at some \(t > \tau^*(\lambda_G^2)\).
not exiting at some \( t \in [\tau^*(\lambda_2^G), \tilde{t}] \) reveals that firm 2 has observed its last customer no earlier than \( t - \tau^*(\lambda_2^G) \); otherwise its belief would have fallen below \( \pi^*(\lambda_2^G) \) at some time before \( t \). Hence, firm 2 not exiting at \( t \in [\tau^*(\lambda_2^G), \tilde{t}] \) also reveals that \( \omega_{t-\tau^*(\lambda_2^G)} = G \). Consequently, from the point of view of firm 1, the lack of customers in \( [0, t - \tau^*(\lambda_2^G)] \) becomes irrelevant as far as its belief about the prevailing state is concerned. Intuitively, firm 1 knows that firm 2 has “fresher” news and can discard part of the information contained in its private history.

### 4.3 Equilibrium Uniqueness

In general, the equilibrium identified in Section 3.2 is not the unique one. Roughly, if firms’ customer arrival rates are sufficiently similar, there also exists an equilibrium in which the firm with the smaller first exit time \( \tau^*(\lambda_i^G) \) exits first with probability one. For example, for the parameters in Figure 1, both \( (\sigma_0^1, \sigma_2^*(\lambda_2^G)) \) and \( (\sigma_1^*(\lambda_1^G), \sigma_0^2) \) are equilibria of the game. In fact, if firm 1 plays the strategy \( \sigma_0^1 \), at any time before \( \tau^*(\lambda_1^G) \), firm 2 has incentives to remain in business because its continuation payoff is strictly positive. In other words, firm 2, anticipating that firm 1 will eventually exit, is willing to remain in the market at beliefs below the cutoff \( \pi^*(\lambda_1^G) \).

Interestingly, on the path induced by the equilibrium \( (\sigma_1^*(\lambda_1^G), \sigma_0^2) \), before \( \tau^*(\lambda_1^G) \), the continuation payoff of firm 2 is sometimes non-monotone in its posterior belief. As illustrated in Figure 3, a firm’s process of customer arrival affects its second-order belief, and hence the expected exit time of the rival.

When payoff externalities are absent, in light of Rosenberg et al. (2007), it is natural to expect all the equilibria to be in cutoff strategies. The non-monotonicity of the continuation payoff as well as the results by Murto (2004) suggest that this is not true in our setup. It is easy to construct simple mixed strategy equilibria. For example, for some parameters, the following strategy profile is an equilibrium. Firm 2 exits with positive probability at time \( \tau^*(\lambda_2^G) \) whenever \( N_{\tau^*(\lambda_2^G)}^2 = 0 \). If it does not exit at that time, it never exits ever after. Firm 1 adopts the strategy \( \sigma_2^1_{\pi^*(\lambda_1^G)} \). The probability with which firm 2 exits is chosen such that firm 1’s best-reply to it makes firm 2 indifferent between exiting at \( \tau^*(\lambda_2^G) \) and never exiting ever after along the history with no customers.
Nevertheless, we are able to show that under appropriate conditions there exists a unique strategy profile that survives iterated deletion of conditionally dominated strategies.

**Theorem 2.A.** Assume that \((\lambda_B^1 + \lambda_B^2)R - c \leq 0\). In the case of both conclusive and inconclusive news, i.e., \(\lambda_B \geq 0\), for any \(\lambda_G^1\), there exists a \(\lambda_G^2 > \lambda_G^1\) such that for \((\lambda_G^1, \lambda_G^2)\), \(\lambda_G^2 > \lambda_G^1\), \((\sigma_0^1, \sigma_{\pi^*}(\lambda_G^2))\) is the unique strategy profile that survives iterated deletion of conditionally dominated strategies; hence, in the unique equilibrium, the larger firm exits from the industry (with probability one).

**Theorem 2.B.** In the case of conclusive news, i.e., \(\lambda_B = 0\), there exists an open set of pairs \((\lambda_G^1, \lambda_G^2)\), \(\lambda_G^2 < \lambda_G^1\), under which \((\sigma_0^1, \sigma_{\pi^*}(\lambda_G^2))\) is the unique strategy profile that survives iterated deletion of conditionally dominated strategies, provided that \(R\) is high enough (or \(c\) is low enough) and that \(r\) and \(\gamma\) are high enough; hence, in the unique equilibrium, the smaller firm exits from the industry (with probability one).

Intuitively, the lower \(\tau^*(\lambda_G^2)\) is, the more influential firm 2’s action, that is, the stronger the inference drawn by firm 1 by observing firm 2 not exiting the market. In other words, the lower \(\tau^*(\lambda_G^2)\) is, the more firm 1 benefits from observational learning. Because observing firm 2 not exiting brings good news, this strengthens incentives for firm 1 to remain in the market. As a consequence, if this “signaling disadvantage” is large enough, in the unique equilibrium, firm 1 eventually becomes the monopolist.

To put it differently, private learning generates a discouragement effect. Firm 2 would be willing to stay in the market at a belief lower than the single-player cutoff only if it expected to eventually become a monopolist. However, by continuing operations firm 2 makes the opponent more optimistic and delays its exit. As a result, anticipating a longer duopoly phase, it is discouraged from remaining in the market.

**Theorem 2** provides sufficient conditions for firm \(i\) to be the first to exit in the unique outcome of the game that survives iterated deletion of conditionally dominated strategies. Interestingly, depending on the parameters the larger or the smaller firm eventually becomes the monopolist.

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20As shown in the Appendix, when \((\lambda_B^1 + \lambda_B^2)R - c > 0\), the statement is slightly weakened in that, if \(r > \gamma + \lambda_B\), we can still identify a set of pairs \(L \in (c/R, \infty) \times (c/R, \infty)\), \(\lambda_G^2 > \lambda_G^1\), for which \((\sigma_0^1, \sigma_{\pi^*}(\lambda_G^2))\) is the unique equilibrium, but in contrast to **Theorem 2.A**, \(\text{proj}_1 L \neq (c/R, \infty)\).

21While **Theorem 2** gives sharp predictions, the sufficient parametric conditions we provide are somewhat not tight. We believe that the uniqueness results extend to a larger set of parameters but we were unable to derive tighter bounds.
Our results provide a rationale for why the technological features of an industry affect the relationship between firm size and the likelihood of survival, as shown by Agarwal and Audretsch (2001) (see also Lieberman, 1990). Agarwal and Audretsch (2001) argue and empirically show that what Geroski (1995) identifies as a stylized fact—that the likelihood of survival is greater for large firms than for small firms—does not hold in the mature phase of the industry life cycle and in high-technology industries.\footnote{Agarwal and Audretsch (2001) explain how the theory of strategic niches can explain this finding. We believe that our model can provide a more compelling justification for the finding, in light of the fact that confidential business information has been recognized to play a key role in market competition (see, for example, the FTC order designed to remedy the anticompetitive effects resulting from Broadcom Limited’s acquisition of Brocade Communications Systems, \url{https://www.ftc.gov/enforcement/cases-proceedings/171-0027/broadcom-limitedbrocade-communications-systems}).}

Figure 4 provides an illustration of the theorem; it identifies the set of pairs \((\lambda_1^G, \lambda_2^G)\) for which firm 2 exits first in the unique strategy profile that survives iterated deletion of dominated strategy. Our model predicts that if there is a high degree of uncertainty, that is, both firms have little information about the market conditions, the small firm exits first. (See right panel in Figure 4.) Intuitively, if firms’ private information is of little use in predicting future profits, in line with Geroski (1995),...
our model predicts that the firm with the lowest profit flow exits first in the unique equilibrium of the game. However, when the large firm has (sufficiently) precise information about the market conditions, it is the small firm that survives the industry decline, as in Agarwal and Audretsch (2001).

As the figure suggests, the set identified in Theorem 2.A is unbounded while the set identified in Theorem 2.B is bounded. That is, whenever the equilibrium is unique, generically, the larger firm exits first. Similarly, there is an asymmetry in the statements of the two parts of the theorem. We discuss these asymmetries below when we discuss the proof.

### 4.3.1 Discussion of the Proof

The proof relies on iterated deletion of (conditionally) dominated strategies. The argument is somewhat involved; here, we illustrate the main ideas. First, we derive a (uniform) lower bound on the belief of firm 1 along any path induced by a strategy profile that survives iterated deletion of conditionally strictly dominated strategies in the case of conclusive news. Next, we use an approximation argument to bound the belief in the case of inconclusive news.

The argument to derive the lower bound on firm 1’s belief is divided in several steps. First, recall that remaining in the market is a dominant action for firm $i$ whenever it attaches a probability higher than the cutoff $\pi^*(\lambda_G^i)$ to the prevailing state being good. (See Lemma 2.)

We show that, under the assumptions of the theorem, for any strategy of firm 2 that prescribes exiting at some belief below $\pi^*(\lambda_G^2)$, firm 2 continuing operation always brings good news. As a result, the belief of firm 1 is bounded away from $\pi^*(\lambda_G^1)$ at any time before $\tau^*(\lambda_G^1)$, making exiting before $\tau^*(\lambda_G^1)$ a dominated action. Second, we argue that if the gap between the two first exit times is large enough, specifically, if $2\tau^*(\lambda_G^2) \ll \tau^*(\lambda_G^1)$, as in Figure 5, at any $t \in [0, 2\tau^*(\lambda_G^2))$, exiting is a dominant strategy for firm 2 whenever its belief falls below the cutoff $\pi^*(\lambda_G^2)$. Intuitively, firm 2 would be willing to remain in the market at a lower belief only if it expected firm 1 to exit with positive probability at some future point in time.

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23 When firm 2 uses a cutoff strategy, this is immediate. However, in contrast to Rosenberg et al. (2007), it is not necessarily true that any rationalizable strategy is a cutoff strategy (see Section 4.3). Hence, arguing that observing that the rival is still active always brings good news requires a more delicate argument.
However, rationality implies that firm 2 does not exit until relatively late in the game, that is, until $\tau^*(\lambda^1_G)$, and it does not find it worthwhile to bear the expected losses to wait until then.

Third, at time $2\tau^*(\lambda^2_G)$, if firm 2 does not exit, firm 1 can infer that the belief of firm 2 never fell below the cutoff $\pi^*(\lambda^2_G)$ in $[0, 2\tau^*(\lambda^1_G))$. In the case of conclusive news, it is easy to identify the pair of firms’ private histories, which are consistent with them playing undominated strategies and with the public history of no exit and which, when combined, would give rise to the lowest posterior belief about the state at time $2\tau^*(\lambda^2_G)$. For firm 1, the “worst” history is the one without any customer. For firm 2, any “worst” history that is consistent with it playing undominated strategies and not exiting by time $2\tau^*(\lambda^2_G)$ involves observing a customer “right after” $\tau^*(\lambda^2_G)$.

(See Figure 5.)

Fourth, combining the inference from these two private histories yields a lower bound to firm 1’s posterior belief at time $2\tau^*(\lambda^2_G)$ along any history on the path generated by a conditionally undominated strategy. For convenience, we further approximate this bound conditioning only on $\omega_{2\tau^*(\lambda^2_G)} = G$ and firm 1 not having observed any customers in $[\tau^*(\lambda^2_G), 2\tau^*(\lambda^2_G))$ to obtain the bound identified with the first red x mark in Figure 5.

Fifth, we can then derive a lower bound on the belief of firm 1 at any time after $2\tau^*(\lambda^1_G)$ using the fact that the posterior belief cannot decrease faster than the private belief, that is, the belief computed while disregarding observational learning. As it turns out, using this bound, we can show that exiting is a dominated action for firm 1 at any time before $\tau^*(\lambda^1_G) + \tau^*(\lambda^2_G)$. (See the red line in Figure 5.)

Last, at time $3\tau^*(\lambda^2_G)$ the same dominance arguments apply, because the problem is stationary. The stationarity hinges on the conclusive news assumptions, as if no firm exits before $3\tau^*(\lambda^2_G)$, it becomes common knowledge that $\omega_{3\tau^*(\lambda^2_G)} = G$. More precisely, for any $n = 3, 4, \ldots$ firm 2 finds it dominant to exit at any $t \in [(n-1)\tau^*(\lambda^2_G), n\tau^*(\lambda^2_G))$ as soon as its belief hits $p^*(\lambda^2_G)$. Therefore, firm 1 finds it dominant not to exit at $t \in [n\tau^*(\lambda^2_G), n\tau^*(\lambda^2_G) + \tau^*(\lambda^1_G))$, irrespective of its private history.

In the case of inconclusive news, i.e., $\lambda_B > 0$, computing the lower bound for the posterior belief is more complicated since it is unclear which pair of firms’ private histories would give rise to the lowest posterior belief. Nevertheless, leveraging the continuity of the posterior belief in $\lambda_B$, we show that for any $\lambda_B > 0$, as $\lambda^2_G$ goes to
infinity, at any time along a history in which firm 2 does not exit, the posterior belief of firm 2 conditional on firm 2 playing undominated strategies is bounded away from the cutoff $\pi^*(\lambda_2 G)$. Intuitively, as $\lambda_2 G$ grows large, firm 1 bases its inference mostly on observational learning, and in the limit the fact that it learns via inconclusive bad news, instead of conclusive bad news, becomes irrelevant.

As noted, the two parts of Theorem 2 are not specular. The key step in the proof is to show that firm 2 finds it dominant to exit at $2\tau^*(\lambda_2 G)$ whenever its belief falls short of the cutoff $\pi^*(\lambda_2 G)$. Now, when the customer arrival rate of firm 2 is sufficiently large, its cutoff belief $\pi^*(\lambda_2 G)$ is arbitrarily low. As a result, in the limit, firm 2 finds it dominant to exit for any $\tau^*(\lambda_1 G) - 2\tau^*(\lambda_2 G)$ is sufficiently large. That is, it is not firm 2’s pessimism about the state of the world that determines its action but rather the amount of time it expects the duopoly to last. The maximum first exit time is increasing in the operating cost $c$. As a result, if firm 2 is sufficiently impatient and the operating cost is such that the first exit time of firm 1 is sufficiently high, firm 2 will find it dominant to exit at $2\tau^*(\lambda_2 G)$.

To be clear, the first part of Theorem 2 is valid for any $\lambda_B > 0$. We expect that the second part is valid for sufficiently low $\lambda_B$.\footnote{To be clear, the first part of Theorem 2 is valid for any $\lambda_B > 0$. We expect that the second part is valid for sufficiently low $\lambda_B$.}
4.4 Extensions

Other Asymmetries. Other types of asymmetries can easily be accommodated. Specifically, our results hold true if firms are asymmetric in their cost of operation ($c$), their revenues ($R$), their discount rate ($r$), or their rate of arrival of customers in the bad state ($\lambda_B$). More formally, for any set of parameters, $(c^1, c^2, R^1, R^2, r^1, r^2, \lambda_B^1, \lambda_B^2)$ and any $\lambda_G^1$, firm 2 exits first in the unique equilibrium of the game provided that $\lambda_G^2$ is large enough. In addition, for any set of parameters, $(c^1, c^2, R^1, R^2, r^1, r^2, \lambda_B^1, \lambda_B^2)$, there exists an open set of pairs $(\lambda_G^1, \lambda_G^2)$ such that $\lambda_G^2 < \lambda_G^1$ and firm 2 exits first in the unique equilibrium of the game provided that $R^1/c^1$ and $r^2$ are high enough.

In a setup with asymmetric primitive parameters, the following comparative statics result is almost immediate.

**Proposition 2.** For any set of parameters $(c^1, c^2, R^1, R^2, r^1, r^2, \lambda_B^1, \lambda_B^2)$, the set of pairs $(\lambda_G^1, \lambda_G^2)$ identified in Theorem 2 is increasing (in the inclusion order) in $r^2$ and $c^2$.

We believe also that the set of strategy profiles that survive iterated deletion of dominated strategy should be increasing in $r^2$ and $c^2$. Intuitively, for any strategy of firm 1, the expected continuation payoff of firm 2 along any history is decreasing in $c^2$ and $r$. Hence, in the iterated procedure, whenever along some history exiting is dominant for firm 2 in a game in which the operating cost of firm 2 is $c^2$ (the discount rate of firm 2 is $r^2$), exiting along that history is also dominant in a game in which the operating cost of firm 2 is $c^2'$ or $r^2'$ (the discount rate of firm 2 is $r^2'$). Deleting more strategies for firm 2 among those that prescribe remaining in the market along some history, makes firm 1 more optimistic and can only increase the histories along which remaining in the market is dominant. While intuitive, this argument is difficult to formalize unless one focuses on a specific deletion procedure, as we do in Proposition 2.

Numerical simulations suggest that the comparative statics with respect to the discount rate also holds in a setup with symmetric primitive parameters. (See also Figure 4.)

Public Information. Our results are robust to the introduction of background public learning. For example, in the presence of public conclusive good news, as long as the rate of arrival of public news is low enough, if the customer arrival rate of
firm 2 is sufficiently high, in the unique equilibrium, firm 2 exits first with probability one. On the one hand, along any history, the additional information from the public news is of little help to firm 2 in drawing an inference about the state of the world. On the other hand, if the informativeness of the public signal is sufficiently low, the bad news from the absence of public good news cannot offset the good news from firm 2 remaining in the market. As a result, our dominance argument holds true. However, as private learning plays a key role in our proof, a sufficiently informative public signal may overturn our result by weakening the role of signaling.

Alternately, consider the case in which the state is publicly revealed at the jump times of a (state-independent) Poisson process \((P(t))_{t \geq 0}\). In this case, as soon as the public signal reveals that the market has become unprofitable, the game enters a war of attrition phase, as in Section 4.1. However along the path with no bad conclusive news, the same analysis applies. Again, as long as the intensity of the Poisson process is low enough, the uniqueness results hold with no substantial changes.

Other Signal Structures. While a complete analysis under an alternative signal structure is beyond the scope of this paper, it is worth noting that the good news assumption is crucial only insofar as it allows us to use a recursive argument to prove uniqueness. A mixed-news conclusive signal structure, as the one above, would preserve this property, and, as long as the informativeness of the private signal is proportional to the expected profit flow, the first exit time would be single-peaked.

Reversible Exit. Allowing for reversible exit complicates the equilibrium analysis, as firms now have access to a richer action space to try and signal their private information. The full characterization of equilibria seems out of reach. However, whenever either of the two pure strategy candidates above is an equilibrium of the game with irreversible exit, it remains a Perfect Bayesian equilibrium of the game with reversible exit.

In fact, if news is inconclusive and being a monopolist is always profitable regardless of the state of the world, signaling plays a limited role, and regardless of the specification of the beliefs off the path, there exists an equilibrium in which the weaker firm exits first with probability one. If instead a monopolist incurs a loss whenever the industry is unprofitable, one can specify the (off-path) posterior of the
stronger firm after re-entry of the rival to be sufficiently high to deter its exit and sustain the equilibrium behavior.

5 Interpretation as an Investment Game

While the main body of the paper focuses on exit from a declining industry, in this section we show that our model can be used to analyze entry decisions in the presence of private learning. As explained below, the equivalent entry model can offer insights into the dynamics of investment in a disruptive technology (Christensen, 1997). The section is written in a relatively informal way, as the objective is to illustrate the versatility of our model.

In the entry game, two firms operating in an established market have the option to irreversibly enter a market of unknown profitability (and simultaneously exit the established market). Operating in the new market yields to firm $i$ a flow profit $\pi_{\omega_t}^i$, irrespective of the market structure, where $\omega_t$ denotes the profitability of the new market. The idea is that in the emerging market, competition is less fierce: either the new market can accommodate both firms producing at capacity, or neither of them. When two firms operate in the established market, each of them earns a profit $D^i$. Firm $i$ obtains a profit $M^i$ from being the monopolist in the established market, $M^i > D^i$. As usual, firms discount payoff at rate $r$.

The profitability of the new market evolves over time. It takes two values, $\omega_t \in \{G, B\}$. Initially, both firms attach probability one to the market being unprofitable, $\omega_0 = B$. In line with this interpretation, let $\pi_G^i > \pi_B^i$ and $\pi_G^i > D^i > \pi_B^i$, so that entering the new market is ever optimal. Conditional on being initially unprofitable, the market irreversibly becomes profitable, unbeknown to the two firms, at some random time that is exponentially distributed with parameter $\gamma > 0$. Before entering the new market, each firm learns about the quality of the market from conclusive bad news signals with intensity $\lambda_{\omega_t}^i$, where $\lambda_B^i > \lambda_G^i = 0$, $i = 1, 2$.

**Assumption 2.** The followings hold for $i = 1, 2$

$$
\pi_G^i = D^i + \frac{\kappa}{\lambda_B^i}, \quad \pi_B^i = \frac{\kappa}{\lambda_B^i}, \quad M^i = D^i + \frac{\eta}{\lambda_B^i},
$$

where $\eta > \kappa > 0$. 

24
To interpret the model and the assumption, consider the decision to invest in a disruptive technology. For example, minicomputers or desktop personal computers represented a disruptive technology to the mainframe computer producers in the seventies and eighties; similarly, the analogue photography industry was eventually destroyed by digital imagery. In this context, Assumption 2 implies that the firm that is better at spotting disruptive opportunities is also the one that is better positioned in the established market. In line with the celebrated “innovator’s dilemma,” (Christensen, 1997) investing in the disruptive technology comes at a higher cost for the leader in the mainstream market because it cannibalizes its existing, profitable business. Nevertheless, the leader in the mainstream market may have superior information about the disruptive technology. Case in point: Kodak developed and patented many of the components of digital imaging technology but failed to make the transition from film to digital photography.26

While restrictive, the assumption ties together a firm’s learning speed to its gain from investing in the new market.27 We want to show that this entry game is “equivalent” to the exit game with parameters \( \lambda_G^i = \lambda_B^i, \lambda_B = 0, R^i = D^i, \) and \( c = \kappa. \) In fact, in the entry model, given a strategy profile \((\sigma^1, \sigma^2)\), the payoff of firm \( i \) can be written as:

\[
E^{(\sigma^1, \sigma^2)} \left[ \int_0^{\sigma_i} e^{-rt} \left( D^i + 1_{\{\sigma_j < t\}}(M^i - D^i) \right) dt + \int_{\sigma_i}^{\infty} e^{-rt} \pi^i_{\omega t} dt \right].
\]

Subtracting the constant term \( E \left[ \int_0^{\infty} e^{-rt} (\pi^i_{\omega t}) dt \right] \), we obtain

\[
E^{(\sigma^1, \sigma^2)} \left[ \int_0^{\sigma_i} e^{-rt} \left( (D^i - \pi^i_{\omega t}) + 1_{\{\sigma_j < t\}}(M^i - D^i) \right) dt \right].
\]

Using the relationships above to replace the parameters of the exit game for the parameters of the entry game, the integral above is almost proportional to (1). The difference is that in the exit game, the payoff after the rival acts depends on the underlying state of the world, which is not true for the entry game. However, the

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27The assumption reduces the degrees of freedom in choosing the parameters of the entry model. It simplifies the proof of the equivalence between this game and the game in the exit game, but we believe that it could be relaxed.
behavior of a firm after its competitor exits plays a limited role in the analysis. In fact, it is almost immediate that Theorem 1 and Theorem 2.B generalize to this setup. Theorem 2.A holds in a slightly weaker form: for any set of parameters, there exists a set of pairs \((\lambda_2^G, \lambda_B), \lambda_B^G > \lambda_2^G\), under which, in the unique equilibrium, firm 2 invests first with probability one.\(^{28}\)

With this interpretation, our model predicts that when the prospects of the disruptive technology are sufficiently uncertain, the established firm fails to invest in it in a timely manner. On the other hand, the model predicts that a firm with a large informational comparative advantage will successfully seize the opportunity to establish itself in the new market. For example, IBM, which dominated the mainframe market, correctly assessed that it was worth investing in the PC market (see, for example, Christensen, 1997, Chapter 5).

Our entry model is certainty too parsimonious to describe the decision to invest in a disruptive technology, as it cannot account, for example, for the possibility of acquiring a competitor. Yet, the vast literature on disruptive technology seems to have overlooked the role of observational learning, which, as shown by our model, may affect the investment dynamics.

6 Conclusions

We analyze a dynamic model of exit from a stochastically declining market. We investigate how private learning affects the equilibrium dynamics. We provide sufficient conditions under which the equilibrium is unique. When the equilibrium is unique, the weaker firm exits first with probability one. We show that in our model, the strength of a firm is determined by its first exit time, a measure of its signaling disadvantage. Crucially, the first exit time is non-monotonic in the firm size or, equivalently, in the learning speed. As a result, our model provides a novel explanation, based on informational externalities, of the fact that in some industries, smaller firms survive the decline. Specifically, our paper offers a testable theory of exit that ties the industry economics primitives to exit dynamics. Furthermore, we show that the model can offer insight into the dynamics of entry in new market.

We conjecture that our proof technique can be applied to other asymmetric timing games with private learning, potentially with more than two players. For example,\(^{28}\) For the formal statements, see the Online Appendix.
the recursive dominance argument can be adapted to show uniqueness in some asymmetric preemption games with evolving state and private learning by identifying an appropriate bound on a player’s continuation payoff if he does not act at the single-agent optimal cutoff.\textsuperscript{29}

Among the features missing from our model, we believe that two are particularly important. First, our stylized model abstracts away from pricing decisions. In reality, as shown by Sweeting et al. (2019), firms may want to choose their prices to signal their private information about the market profitability. We are not aware of pricing models with two-sided private learning about a common underlying state of the world.\textsuperscript{30} Second, our results do not immediately extend a setup in which the market profitability fluctuates over time. This generalization may have the potential to provide a novel model of shakeouts. We leave these questions for future research.

\textsuperscript{29}Thomas (2020) proves equilibrium uniqueness in two-player preemption games with private learning (and perfectly persistent state) using a different technique.

\textsuperscript{30}The paper by Sweeting et al. (2019) is an exception: they extend their finite-horizon model to allow for this possibility.
References


A Omitted Proofs

A.1 Proof for Section 4

Proof of Lemma 1. If $\lambda_{iG} R \leq c$, even when the prevailing state is good, firm $i$’s expected flow payoff is non-positive. Hence, the firm finds it optimal to exit immediately, i.e., $\pi^*(\lambda_{iG}) = 1$. To prove the converse, we show that if $\lambda_{iB} = 0$, whenever $\lambda_{iG} R > c$, $\pi^*(\lambda_{iG}) < 1$. A fortiori $\pi^*(\lambda_{iG}) < 1$ when $\lambda_{iB} > 0$.

When $\lambda_{iG} R > c$, the value function of the best-reply problem solves the following Hamilton-Jacobi-Bellman equation

$$rv(p) = (p\lambda_{iG} + (1 - p)\lambda_{iB})(R + v(j(p)) - v(p)) - c$$
$$- (p(1 - p)(\lambda_{iG} - \lambda_{iB}) + p\gamma) v'(p),$$

where

$$j(p) = \frac{p\lambda_{iG}}{p\lambda_{iG} + (1 - p)\lambda_{iB}},$$

denotes the belief after observing a customer.

In the case of conclusive news, i.e., $\lambda_{iB} = 0$, by solving the Hamilton-Jacobi-Bellman equation, we find that in the continuation region,

$$v(p) = -\left(1 - \frac{\lambda_{iG}}{\gamma + \lambda_{iG} + r} p\right) \frac{c}{r} + \frac{\lambda_{iG}}{\gamma + \lambda_{iG} + r} p(R + v(1)) + p\Omega(p)\left(\frac{c + \frac{\lambda_{iG}}{\gamma + \lambda_{iG} + r} R}{1 + \frac{c + \frac{\lambda_{iG}}{\gamma + \lambda_{iG} + r} R}{\gamma + \lambda_{iG} + r}}\right) C,$$

where $C$ is a constant of integration. Since

$$v(1) = -\left(\frac{\gamma + r}{\gamma + \lambda_{iG} + r}\right) \frac{c}{r} + \frac{\lambda_{iG}}{\gamma + \lambda_{iG} + r} (R + v(1)) + \gamma \left(\frac{c + \frac{\lambda_{iG}}{\gamma + \lambda_{iG} + r} R}{1 + \frac{c + \frac{\lambda_{iG}}{\gamma + \lambda_{iG} + r} R}{\gamma + \lambda_{iG} + r}}\right) C,$$

we have

$$C = \gamma^{-1 - \frac{c}{\gamma + r}} \left(\frac{\gamma + r}{\gamma + \lambda_{iG} + r}\right) \left(v(1) + \frac{c}{r}\right) - \frac{\lambda_{iG}}{\gamma + \lambda_{iG} + r} R.$$

Using smooth-pasting at the cutoff $\pi^*(\lambda_{iG})$, $v'(\pi^*(\lambda_{iG})) = 0$,

$$v(1) = -\frac{c}{r} + \frac{\lambda_{iG}}{\gamma + r} R$$
$$+ \frac{\gamma + \lambda_{iG} + r}{\gamma + r} \frac{\pi^*(\lambda_{iG})\lambda_{iG} \gamma}{(\gamma + r)(\pi^*(\lambda_{iG})\lambda_{iG} + r)\left(\frac{\Omega(\pi^*(\lambda_{iG}))}{\gamma}\right)^\gamma} - \pi^*(\lambda_{iG})\lambda_{iG} \gamma R.$$
where
\[
\Omega_i(p) = \frac{\gamma + (1 - p)\lambda_i^G}{p}.
\]

Using this equation to replace \(v(1)\) in (3), we obtain
\[
C = \gamma - \frac{\pi^*(\lambda_i^G)\lambda_i^G\gamma}{(\gamma + r)(\pi^*(\lambda_i^G)\lambda_i^G + r)(\frac{\Omega^*(\lambda_i^G)}{\gamma})^{\gamma + \lambda_i^G} - \pi^*(\lambda_i^G)\lambda_i^G\gamma}R,
\]

which, replaced in the value matching condition, \(v(\pi^*(\lambda_i^G)) = 0\),
\[
- \left(1 - \frac{\lambda_i^G}{\gamma + \lambda_i^G + r}\pi^*(\lambda_i^G)\right)\frac{c}{r} + \frac{\lambda_i^G}{\gamma + \lambda_i^G + r}\pi^*(\lambda_i^G) (R + v(1))
\]

\[+ \pi^*(\lambda_i^G) \Omega^*(\lambda_i^G) \frac{1 + \frac{c}{r^{\gamma + \lambda_i^G}}}{C = 0},\]

yields, after some manipulations,\(^{31}\)
\[
\Omega_i(\pi^*(\lambda_i^G)) = \frac{\lambda_i^G \gamma + \lambda_i^G(\gamma + \lambda_i^G + r) R}{c} - \frac{\lambda_i^G \gamma + \lambda_i^G + r}{r}
\]

\[+ \frac{\gamma \lambda_i^G(\gamma + \lambda_i^G)}{r(\gamma + r)(\Omega_i(\pi^*(\lambda_i^G))/\gamma)^{r/(\gamma + \lambda_i^G)}}.\]

The right-hand side is decreasing in \(\pi^*(\lambda_i^G)\), and the left-hand side is increasing in \(\pi^*(\lambda_i^G)\). Hence, there exists at most one root to (4). At \(\pi^*(\lambda_i^G) = 1\), \(\lambda_i^G R > c\) implies that the left-hand side if smaller than the right-hand side, while in the limit at \(\pi^*(\lambda_i^G)\) goes to zero, the opposite is true. It follows that there exists a unique root \(\pi^*(\lambda_i^G) < 1\).

Optimality follows by standard verification arguments. (See, for example Øksendal and Sulem, 2019, Theorem 3.2.).

In the general case, i.e. \(\lambda_B \geq 0\) the existence and uniqueness results for functional differential equations guarantee that there exists a unique twice continuously differentiable solution to the Hamilton-Jacobi-Bellman equation given an initial guess for \(v(1)\); see for example Corduneanu et al., 2016, Theorem 2.4.\(^{32}\) Define the mapping \(\Gamma : [0, (\lambda_i^G R - c)/r] \rightarrow \mathbb{R}\), which maps an initial guess \(v(1)\) to the following function of the corresponding solution \(\min_{p \in [0,1]} v(p) + |v'(p)|\). Notice that \(\Gamma(0) = (\lambda_i^G R - c)/r\), while \(\Gamma((\lambda_i^G R - c)/r) < 0\). By Corduneanu et al., 2016, Theorem 3.6, the mapping \(\Gamma\) is continuous. Hence, by the intermediate value theorem, there exists a guess such

\(^{31}\)It can be checked that when \(\gamma = 0\), (4) reduces to the cutoff characterization of Décamps and Mariotti (2004) and Keller et al. (2005).

\(^{32}\)After a change of variable \(q = 1 - p\), the functional differential equation (2) is a Volterra operator. After bounding the domain to \(q \in [0,1 - \varepsilon)\), for arbitrarily small \(\varepsilon > 0\), the assumptions of Corduneanu et al., 2016, Theorem 2.4 are satisfied.
that the solution to the Hamilton-Jacobi-Bellman equation satisfies \( v(p) = v'(p) = 0 \) for some \( p \in (0,1) \). Again, optimality and uniqueness follow by standard verification arguments.

Using the value matching and smooth-pasting conditions, it is easily verified that the optimal cutoff \( \pi^*(\lambda_G^i) \) satisfies the following equation

\[
\pi^*(\lambda_G^i) = \frac{c}{(\lambda_G^i - \lambda_B) (R + v(j(\pi^*(\lambda_G^i))))} - \frac{\lambda_B}{\lambda_G^i - \lambda_B}.
\] (5)

Given an increasing function \( v \), the equation has at most one solution \( \pi^*(\lambda_G^i) \). The right-hand side is decreasing in \( v \), in \( j \), and in \( \lambda_G^i \). Both the value function \( v \) and the function \( j \) are increasing pointwise in \( \lambda_G^i \). Hence, by the implicit function theorem, we can conclude that the optimal cutoff \( \pi^*(\lambda_G^i) \) is decreasing in \( \lambda_G^i \).

(ii) In an abuse of notation, in the general case, i.e., \( \lambda_B \geq 0 \), we write

\[
\Omega(p) = \frac{\gamma + (1-p)(\lambda_G^i - \lambda_B)}{p}.
\]

First, as proved above, \( \pi^*(\lambda_G^i) \) is decreasing in \( \lambda_G^i \). Second, notice that

\[
\tau^*(\lambda_G^i) = \frac{\ln (\Omega(\pi^*(\lambda_G^i))/\gamma)}{\gamma + \lambda_G^i - \lambda_B}.
\]

It follows from (5) that

\[
\pi^*(\lambda_G^i) \geq \frac{c}{(\lambda_G^i - \lambda_B) (R + (\lambda_G^i R - c)/r)} - \frac{\lambda_B}{\lambda_G^i - \lambda_B} =: \pi(\lambda_G^i),
\]

where the bound is derived by replacing \( v(j(\pi^*(\lambda_G^i))) \) with \((\lambda_G^i R + R/2 - c)/r\). Clearly, \( v(j(\pi^*(\lambda_G^i))) < (\lambda_G^i R + R/2 - c)/r \) because \( \gamma > 0 \) and the state market conditions eventually deteriorate.

By de l’Hôpital’s rule

\[
\lim_{\lambda_G^i \to \infty} \frac{\ln (\Omega(\pi(\lambda_G^i))/\gamma}{\gamma + \lambda_G^i - \lambda_B} = \lim_{\lambda_G^i \to \infty} \frac{1}{\Omega(\pi(\lambda_G^i))} \left( \frac{1 - \pi(\lambda_G^i)}{\pi(\lambda_G^i)} + \frac{\gamma + \lambda_G^i - \lambda_B - \pi'(\lambda_G^i)}{\pi(\lambda_G^i)^2} \right)
= \lim_{\lambda_G^i \to \infty} \frac{1 - \pi(\lambda_G^i)}{\gamma + (1 - \pi(\lambda_G^i))(\lambda_G^i - \lambda_B)} + \frac{\gamma + \lambda_G^i - \lambda_B}{\gamma + (1 - \pi(\lambda_G^i))(\lambda_G^i - \lambda_B)} \pi(\lambda_G^i).
\]

\[\text{To be precise, abusing notation let } v(p, \lambda_G^i) \text{ and } j(p, \lambda_G^i) \text{ be the optimal value function and the function that describes the belief after the arrival of a customer respectively, when the rate of arrival of consumers in the good state is } \lambda_G^i. \text{ Then, } v(p, \lambda_G^i) \text{ is continuously differentiable in } p, \text{ and } j(p, \lambda_G^i) \text{ is continuously differentiable in } \lambda_G^i. \text{ By the envelope theorem of Milgrom and Segal (2002), the value function } v(p, \lambda_G^i) \text{ is continuously differentiable in } \lambda_G^i.
Since \( \pi'(\lambda_G^i) = O((\lambda_G^i)^{-2}) \) and \( \pi(\lambda_G^i) = O((\lambda_G^i)^{-1}) \), the right-hand side converges to zero. Hence,

\[
0 = \lim_{\lambda_G^i \to \infty} \frac{\ln (\Omega(\pi(\lambda_G^i))/\gamma)}{\gamma + \lambda_G^i - \lambda_B} \geq \lim_{\lambda_G^i \to \infty} \frac{\ln (\Omega(\pi^*(\lambda_G^i))/\gamma)}{\gamma + \lambda_G^i - \lambda_B},
\]

and \( \lim_{\lambda_G^i \to \infty} \tau^*(\lambda_G^i) = 0 \). We now state and prove two lemmas that are used later.

**Lemma 3.** The following holds,

\[
\lim_{\lambda_G^i \to \infty} \pi^*(\lambda_G^i) = 0.
\]

**Proof.** First, in the case of conclusive news, i.e., \( \lambda_B = 0 \), smooth-pasting implies

\[
0 = \lambda_G^i \pi^*(\lambda_G^i)(R + v(1)) - c.
\]

Moreover,

\[
v(1) \geq \frac{\lambda_G^i (1 - e^{-(\gamma + \lambda_G^i + r)\tau})}{\gamma + r + \lambda_G^i e^{-(\gamma + \lambda_G^i + r)\tau}} R
- \frac{(\gamma + r)(\lambda_G^i + \gamma) - \lambda_G^i re^{-(\gamma + \lambda_G^i + r)\tau} + \gamma (r + \gamma + \lambda_1)e^{-r\tau}}{r(\gamma + \lambda_G^i)(\gamma + r + \lambda_G^i e^{-(\gamma + \lambda_G^i + r)\tau})} c
\]

for all \( \tau \geq 0 \). Hence, \( v(1) \to \infty \) as \( \lambda_G^i \to \infty \). As a result, \( \lim_{\lambda_G^i \to \infty} \lambda_G^i \pi^*(\lambda_G^i) = 0 \).

Because the cutoff belief in the case of conclusive news is an upper bound to the cutoff belief in the case of inconclusive news, it follow that the result generalizes to the case of \( \lambda_B > 0 \).

**Lemma 4.**

(i) \( \lim_{\lambda_G^i \to \infty} \pi''(\lambda_G^i) = 0 \).

(ii) \( \lim_{\lambda_G^i \to \infty} \pi'''(\lambda_G^i) = 0 \).

**Proof.** (i) As in footnote 33, we let \( v(p, \lambda_G^i) \) and \( j(p, \lambda_G^i) \) be the optimal value function and the function that describes the belief after the arrival of a customer, respectively, when the rate of arrival of consumers in the good state is \( \lambda_G^i \). By the implicit function
Lemma 5. Consider two sets of parameters \((c, R, r, \lambda_G^i, \lambda_B, \gamma)\) and \((\hat{c}, R, \hat{r}, \hat{\lambda}_G^i, \hat{\lambda}_B, \hat{\gamma})\) such that

\[
\frac{\hat{\lambda}_G^i}{\hat{\gamma}} = \frac{\lambda_G^i}{\gamma}, \quad \frac{\hat{\lambda}_B}{\hat{\gamma}} = \frac{\lambda_B}{\gamma}, \quad \hat{r} = r, \quad \frac{\hat{c}}{\hat{\gamma}} = \frac{c}{\gamma}.
\]

The optimal value functions and the optimal cutoff beliefs in the two optimal stopping problems coincide.

**Proof.** First, notice that the optimal value function associated with the first optimal stopping problem satisfies the Hamilton-Jacobi-Bellman equation of the second; see (2). In addition, the smooth pasting and value matching conditions associated with the two optimal stopping problems are identical. Since a standard verification theorem
applies (see Øksendal and Sulem, 2019, Theorem 3.2), the optimal value functions and the optimal cutoff beliefs in the two optimal stopping problems coincide.

Throughout, we fix a vector \((c, R, r, \lambda_B)\). In light of Lemma 5, we consider the optimal stopping problem “scaled” by some \(\gamma\), that is, we should consider how the first exit time changes with \(\lambda_G\) in the optimal stopping problem parametrized by \((\gamma c, R, \gamma r, \gamma \lambda_G, \gamma \lambda_B, \gamma)\), where the first element is the flow cost of remaining into business.

Define \(\hat{\tau}^*(\lambda_G)\) to be the optimal cutoff when \(\gamma = 1\), and the other parameters are \((c, R, r, \lambda_G, \lambda_B)\). By construction, the optimal cutoff associated with the decision problem parametrized by \((\gamma c, R, \gamma r, \gamma \lambda_G, \gamma \lambda_B, \gamma)\) is also \(\hat{\tau}^*(\lambda_G)\). Define

\[
\hat{\tau}^*(\Lambda_G; \gamma) = \frac{\ln \left( \frac{1 - \hat{\pi}^*(\Lambda_G)}{\hat{\pi}^*(\Lambda_G) (\Lambda_G - \lambda_B)} \right)}{\gamma(1 + \Lambda_G - \lambda_B)}
\]

so that the first exit time associated with the decision problem parametrized by \((\gamma c, R, \gamma r, \gamma \lambda_G, \gamma \lambda_B, \gamma)\) is equal to \(\hat{\tau}^*(\Lambda_G; \gamma)\). To say it differently, for any set of parameters, \(\tau^*(\lambda_G) = \hat{\tau}^*(\lambda_G/\gamma; \gamma)\). Clearly, proving the single-peakedness of \(\hat{\tau}^*\) is equivalent to proving the single-peakedness of \(\tau^*\).

By differentiation,

\[
\hat{\tau}^{*'}(\Lambda_G; \gamma) = \frac{1}{\gamma(1 + \Lambda_G - \lambda_B)} \left( -\hat{\tau}^*(\Lambda_G; \gamma) + \frac{1 - \hat{\pi}^*(\Lambda_G)}{1 + (1 - \hat{\pi}^*(\Lambda_G))(\Lambda_G - \lambda_B)} - \frac{1 + \Lambda_G - \lambda_B}{(1 + (1 - \hat{\pi}^*(\Lambda_G))(\Lambda_G - \lambda_B)) \hat{\pi}^*(\Lambda_G)} \hat{\tau}^{*'}(\Lambda_G) \right)
\]

We now define

\[
\psi(\Lambda_G) := \frac{1 - \hat{\pi}^*(\Lambda_G)}{1 + (1 - \hat{\pi}^*(\Lambda_G))(\Lambda_G - \lambda_B)} - \frac{1 + \Lambda_G - \lambda_B}{(1 + (1 - \hat{\pi}^*(\Lambda_G))(\Lambda_G - \lambda_B)) \hat{\pi}^*(\Lambda_G)} \hat{\tau}^{*'}(\Lambda_G).
\]

Notice that \(\psi(\Lambda_G)\) does not depend on \(\gamma\). The following lemma establishes two properties of the function \(\psi(\Lambda_G)\), which we use later.

**Lemma 6.**

(i) \(\psi'(\Lambda_G) < 0\) a.e. for \(\Lambda_G \geq \Lambda^i_G\), for some \(\Lambda^i_G \in (c/R + 1/2, \infty)\).

(ii) \(\inf_{\Lambda_G \in (c/R + 1/2, \Lambda^i_G]} \psi'(\Lambda_G) > 0\).
Proof. For (i), differentiating yields

\[
\psi'(\Lambda_G^i) = -\left( \frac{1}{1 + (1 - \hat{\pi}^*(\Lambda_G^i))(\Lambda_G^i - \lambda_B)} \right)^2 \\
\left( (1 - \hat{\pi}^*(\Lambda_G^i))^2 + 2\hat{\pi}''(\Lambda_G^i) + (1 + \Lambda_G^i - \lambda_B)(1 + (1 - 2\hat{\pi}^*(\Lambda_G^i))(\Lambda_G^i - \lambda_B)) \left( \frac{\hat{\pi}''(\Lambda_G^i)}{\hat{\pi}^*(\Lambda_G^i)} \right)^2 \right) \\
- \frac{1 + \Lambda_G^i - \lambda_B}{\hat{\pi}^*(\Lambda_G^i)(\gamma + (1 - \hat{\pi}^*(\Lambda_G^i))(\Lambda_G^i - \lambda_B))} \hat{\pi}'''(\Lambda_G^i).
\]

Recall that \(\hat{\pi}''(\Lambda_G^i) < 0\) and \(\lim_{\Lambda_G^i \to \infty} \hat{\pi}^*(\Lambda_G^i) = 0\). Moreover, by Lemma 4, for sufficiently high \(\Lambda_G^i\), the terms on the second and third lines are positive a.e. for \(\Lambda_G^i > \Lambda_G^i\) for some \(\Lambda_G^i < \infty\). Because \(\Lambda_G^i < \infty\), (ii) follows.

To conclude, we first show that \(\hat{\tau}^*(\Lambda_G^i; \gamma)\) is single-peaked for sufficiently high \(\gamma\). Observe that \(\hat{\tau}^*(\Lambda_G^i; \gamma)\) is (pointwise in \(\Lambda_G^i\)) decreasing in \(\gamma\). Hence, for sufficiently high \(\gamma\), for all \(\Lambda_G^i \in (c/R + 1/2, \Lambda_G^i]\), \(\hat{\tau}^*(\Lambda_G^i; \gamma) < \inf_{\Lambda_G^i \in (c/R + 1/2, \Lambda_G^i]} \psi(\Lambda_G^i)\). Consequently, for \(\gamma\) high enough, the function \(\hat{\tau}^*(\Lambda_G^i; \gamma)\) is single-peaked as \(\hat{\tau}^*(\Lambda_G^i; \gamma)\) crosses the function \(\psi(\Lambda_G^i)\) no more than once and from below (see (7)). Therefore, \(\hat{\tau}^*(\Lambda_G^i; \gamma)\) must be single-peaked for any \(\gamma\), because a linear transformation of a single-peaked function is single-peaked.

\[ \square \]

Proof of Theorem 1. Here, we provide the proof of Theorem 1 for the case of inconclusive news, i.e., \(\lambda_B > 0\).

With slight abuse of notation, let \(\tau^*(\lambda_G^2, \lambda_B)\) and \(\pi^*(\lambda_G^2, \lambda_B)\) denote the first exit time and the cutoff belief in the benchmark best-reply problem as a function of the arrival rate in the two states. We denote with \(F_t^G: [0, 1] \to [0, 1]\) firm 1’s posterior distribution about firm 2’s belief about the prevailing state at time \(t\), conditional on \(\omega_t = G\) and firm 2’s (private) belief at any point in time \(s \leq t\) having been strictly higher than \(\pi^*(\lambda_G^2, \lambda_B)\). That is

\[
F_t^G(p) := \Pr \left[ \omega_t = G \mid (\gamma_s^2)_{s \leq t} \leq p \right. \\
\left. \omega_t = G, \Pr[\omega_s = G \mid (\gamma_s^2)_{s \leq u}] > \pi^*(\lambda_G^2, \lambda_B), \text{ for all } u \in [0, t] \right].
\]

At any \(t > \tau^*(\lambda_G^2, \lambda_B)\), \(F_t^G\) is absolutely continuous; we denote its density by \(f_t^G\). In the Online Appendix, we prove the following lemmas.
Lemma 7. Fix $\bar{\lambda}_B > 0$. For any $p \in [\pi^*(\lambda_G^2, \lambda_B), 1]$ and any $t > \tau^*(\lambda_G^2, \bar{\lambda}_B)$, the posterior distribution $F_t^G(p)$ and its density $f_t^G(p)$ are Lipschitz continuous in $\lambda_B \in [0, \bar{\lambda}_B]$.

Lemma 8. Fix $\bar{\lambda}_B > 0$. For any $t > \tau^*(\lambda_G^2, \bar{\lambda}_B)$, $p > \pi^*(\lambda_G^2, \lambda_B)$, and $\tau > 0$,

$$\Pr \left[ \Pr[\omega_s = G \mid (N^2_s)_{s \leq u}] > \pi^*(\lambda_G^2, \lambda_B), \text{for all } u \in [t, t + \tau] \right.$$ \hfill
$$\left. \Pr[\omega_t = G \mid (N^2_s)_{s \leq t}] = p, \omega_t = G \right]$$

is Lipschitz continuous in $\lambda_B$.

Define $\varphi_1(1, t) := \Pr[\omega_{s+t} = G \mid \omega_s = G, N^i_s = N^i_{s+t}]$, where $s \geq 0$. In other words, $\varphi_1(p, t)$ is the posterior belief about the prevailing state at time $s + t$ conditional on the state being good at time $s$ and firm $i$ not having observed any customer in $[s, s + t)$.

We want to prove that for sufficiently high $\lambda_G^2$, the strategy profile $(\sigma^1_0, \sigma_{\pi^*(\lambda_G^2, \lambda_B)})$ is an equilibrium of the game. First, by Lemma 1, for sufficiently high $\lambda_G^2$, $2\tau^*(\lambda_G^2, \lambda_B) < \tau^*(\lambda_G^2, \lambda_B)$. Now, along the path induced by this strategy profile, if firm 2 has not exited by time $2\tau^*(\lambda_G^2, \lambda_B)$, the posterior belief about the state at time $\tau^*(\lambda_G^2, \lambda_B)$ is bounded below by

$$\left( \varphi_1(1, \tau^*(\lambda_G^2, \lambda_B)) \right)$$

$$\int^1_{\tau^*(\lambda_G^2, \lambda_B)} \Pr \left[ \Pr[\omega_s = G \mid (N^2_s)_{s \leq u}] > \pi^*(\lambda_G^2, \lambda_B), \text{for all } u \in [\tau^*(\lambda_G^2, \lambda_B), 2\tau^*(\lambda_G^2, \lambda_B)] \right.$$ \hfill
$$\left. \Pr[\omega_{\tau^*(\lambda_G^2, \lambda_B)} = G \mid (N^2_s)_{s \leq t}] = p, \omega_{\tau^*(\lambda_G^2, \lambda_B)} = G \right] dF_{\tau^*(\lambda_G^2, \lambda_B)}(p)$$

$$\cdot \left( \varphi_1(1, \tau^*(\lambda_G^2, \lambda_B)) \right)$$

$$\int^1_{\tau^*(\lambda_G^2, \lambda_B)} \Pr \left[ \Pr[\omega_s = G \mid (N^2_s)_{s \leq u}] > \pi^*(\lambda_G^2, \lambda_B), \text{for all } u \in [\tau^*(\lambda_G^2, \lambda_B), 2\tau^*(\lambda_G^2, \lambda_B)] \right.$$ \hfill
$$\left. \Pr[\omega_{\tau^*(\lambda_G^2, \lambda_B)} = G \mid (N^2_s)_{s \leq t}] = p, \omega_{\tau^*(\lambda_G^2, \lambda_B)} = G \right] dF_{\tau^*(\lambda_G^2, \lambda_B)}(p)$$

$$+ \left( 1 - \varphi_1(1, \tau^*(\lambda_G^2, \lambda_B)) \right) \left( 1 - e^{-\lambda_B \tau^*(\lambda_G^2, \lambda_B)} \right)^{-1}.$$ 

This is a lower bound because the last term at the denominator is an upper bound: firm 2 observing at least one customer in the interval $[\tau^*(\lambda_G^2, \lambda_B), 2\tau^*(\lambda_G^2, \lambda_B))$ is necessary but not sufficient for its private belief to be bounded away from $\pi^*(\lambda_G^2, \lambda_B)$ at any point in time in the interval $[\tau^*(\lambda_G^2, \lambda_B), 2\tau^*(\lambda_G^2, \lambda_B))$. 
By Lemma 7 and Lemma 8 we can identify another bound:

\[
\left( \varphi_1(1, \tau^*(\lambda_G^2, 0)) \right) \int_{\pi^*(\lambda_G^2, 0)}^1 \Pr \left[ \Pr[\omega_s = G \mid (N_s^2)_{s \leq u}] > \pi^*(\lambda_G^2, 0), \text{for all } u \in [\tau^*(\lambda_G^2, 0), 2\tau^*(\lambda_G^2, 0)] \right] \ dF_{\tau^*(\lambda_G^2, 0)}^G(p) - \lambda_B \Delta \\
\cdot \left( \varphi_1(1, \tau^*(\lambda_G^2, 0)) \right) \int_{\pi^*(\lambda_G^2, 0)}^1 \Pr \left[ \Pr[\omega_s = G \mid (N_s^2)_{s \leq u}] > \pi^*(\lambda_G^2, 0), \text{for all } u \in [\tau^*(\lambda_G^2, 0), 2\tau^*(\lambda_G^2, 0)] \right] \ dF_{\tau^*(\lambda_G^2, 0)}^G(p) - \lambda_B \Delta \\
+ \left( 1 - \varphi_1(1, \tau^*(\lambda_G^2, \lambda_B)) \right) \left( 1 - e^{-\lambda_B \tau^*(\lambda_G^2, \lambda_B)} \right)^{-1}
\]

where \( \Delta < \infty \) is a bounded function of the Lipschitz constants in Lemma 7 and Lemma 8. As \( \lambda_G^2 \rightarrow \infty, \tau^*(\lambda_G^2, \lambda_B) \rightarrow 0, \) and the bound converges to 1. If firm 2 has not exited by time \( 2\tau^*(\lambda_G^2, \lambda_B), \) in the limit as \( \lambda_G^2 \rightarrow \infty, \) the posterior belief of firm 1 about the state at time \( \tau^*(\lambda_G^2, \lambda_B) \) is bounded below by \( \varphi_1(1, \tau^*(\lambda_G^2, \lambda_B)) \) by Lemma 1, this inequality holds for sufficiently high \( \lambda_G^2 \). Recall that by Lemma 2, in any equilibrium firm i continues operations as long as its belief is above \( \pi^*(\lambda_G^2) \). As a result, firms’ beliefs at any time before \( \tau^*(\lambda_G^2) \) is uniquely determined by their private history.

Because the same argument applies for any \( n\tau^*(\lambda_G^2, \lambda_B), n \geq 2, \) by Lemma 2, \( \sigma_0^1 \) is a best reply to \( \sigma_{\pi^*(\lambda_G^2, \lambda_B)}^2 \).

Proof of Theorem 2.A. We show that for sufficiently high \( \lambda_G^2, (\sigma_0, \sigma_{\pi^*(\lambda_G^2)}^2) \) is the unique strategy profile that survives iterated deletion of conditionally dominated strategies.

First, assume that \( \lambda_B = 0 \) and \( 2\tau^*(\lambda_G^2) < \tau^*(\lambda_G^1) \). By Lemma 1, this inequality holds for sufficiently high \( \lambda_G^2 \). Recall that by Lemma 2, in any equilibrium firm i continues operations as long as its belief is above \( \pi^*(\lambda_G^2) \). As a result, firms’ beliefs at any time before \( \tau^*(\lambda_G^2) \) is uniquely determined by their private history.

We now argue that regardless of firm 1’s belief about firm 2’s strategy, firm 1’s posterior along the history with no exit is bounded above \( \pi^*(\lambda_G^1) \) at any time before \( \tau^*(\lambda_G^1) \).

To this end, we start by showing that at any time \( t \leq \tau^*(\lambda_G^1) \), for any strategy of firm 2 that survived our first round of deletion, the probability that firm 2’s posterior belief is equal to some \( p \in [0, \pi^*(\lambda_G^1)] \) is higher in bad state than in the good state, provided that \( \lambda_G^1 \) is taken to be arbitrary high. This implies that along the history with no exit, observational learning always brings good news, and firm 1’s private
belief is a lower bound to its posterior belief. The details of the argument are relegated to the only appendix, but here we provide some intuition.

If firm \(1\) expects firm \(2\) to play a cutoff strategy, as in Rosenberg et al. (2007) and Murto and Välimäki (2011), observational learning always brings good news, that is, firm \(2\) continuing operation makes firm \(1\) more optimistic. In fact, the distribution of firm \(2\)’s posterior belief conditional on the good state first-order stochastically dominates the distribution of firm \(2\)’s posterior belief conditional on the bad state. Once one allows for any non-cutoff strategy, this does need to be true. However, in the limit as \(\lambda_2^G\) goes to infinity, two things happen. First, the distribution of firm \(2\)’s posterior beliefs conditional on either states converges to a degenerate distribution concentrated on either \(0\) or \(1\). Second, the range of belief for which exiting is not a dominated action shrinks, since \(\pi^*(\lambda_2^G) \to 0\). As a result, even if firm \(1\) expects firm \(2\) to play a non-cutoff strategy, the probability of firm \(2\) exiting conditional on the prevailing state being bad is higher than the probability of firm \(2\) exiting conditional on the state being good. It follows that exiting before \(\tau^*(\lambda_1^G)\) is a dominated action for firm \(1\) and firm \(2\)’s belief at any time before \(\tau^*(\lambda_1^G)\) is uniquely determined by its private history.

Observe that by the definition of \(\tau^*(\lambda_2^G)\), the integrand is negative; thus the first term is negative. By Lemma 3, as \(\lambda_2^G \to \infty\), \(\pi^*(\lambda_2^G)\lambda_2^G \to 0\). Hence, for sufficiently high \(\lambda_2^G\), the expected continuation payoff is negative. We can then conclude that conditional on not observing any customer in \([0, \tau^*(\lambda_2^G))\), it is dominant for firm \(2\) to exit at time \(\tau^*(\lambda_2^G)\).

Consider now the case in which the posterior belief of firm \(2\) at time \(2\tau^*(\lambda_2^G)\) is again \(\pi^*(\lambda_2^G)\). In this case, the expected continuation continuation payoff from
remaining in the market is bounded above by

$$\int_{2\tau^*(\lambda^2_G)}^{\tau^*(\lambda^2_G)} e^{-r(t-2\tau^*(\lambda^2_G))} \left( \pi^*(\lambda^2_G) e^{-\gamma(t-2\tau^*(\lambda^2_G))} \lambda_G^2 R - c \right) dt$$

$$+ e^{-r(\tau^*(\lambda^2_G)-2\tau^*(\lambda^2_G))} \left( \pi^*(\lambda^2_G) e^{-\gamma(\tau^*(\lambda^2_G)-2\tau^*(\lambda^2_G))} \frac{(\lambda^1_G + \lambda^2_G) R - c}{r} \right).$$

By the same argument as above, for sufficiently high $\lambda^2_G$, the expected continuation payoff is negative and exit is dominant for firm 2. A fortiori, for any $t \in (\tau^*(\lambda^2_G), 2\tau^*(\lambda^2_G))$, it is dominant for firm 2 to exit whenever its belief falls short of $\pi^*(\lambda^2_G)$.

In the case of conclusive news, if firm 1 does not observe an exit at $2\tau^*(\lambda^2_G)$, it is dominant for it not to exit before $\tau^*(\lambda^1_G) + \tau^*(\lambda^2_G)$. In fact, at that time, firm 1 infers that the belief of firm 2 never fell below the cutoff $\pi^*(\lambda^2_G)$ in $[0, 2\tau^*(\lambda^1_G))$. Moreover, in the worst-case scenario, firm 2 observed a customer “right after” $\tau^*(\lambda^2_G)$ (see Figure 5). Consequently, the posterior belief of firm 1 is bounded away from $\pi^*(\lambda^1_G)$ at any time before $\tau^*(\lambda^1_G) + \tau^*(\lambda^2_G)$, and by Lemma 1 remaining in the market is dominant at those times.

To show the desired result, we apply conditional dominance argument recursively. More formally, for any $n = 3, 4, \ldots$, firm 2 finds it dominant to exit at any $t \in [(n-1)\tau^*(\lambda^2_G), n\tau^*(\lambda^2_G))$ as soon as its belief falls short of $\pi^*(\lambda^2_G)$; in fact, for any $n$, the payoff from staying in the market is bounded above by

$$\int_{n\tau^*(\lambda^2_G)}^{(n-1)\tau^*(\lambda^2_G)+\tau^*(\lambda^2_G)} e^{-r(t-n\tau^*(\lambda^2_G))} \left( \pi^*(\lambda^2_G) e^{-\gamma(t-n\tau^*(\lambda^2_G))} \lambda_G^2 R - c \right) dt$$

$$+ e^{-r(\tau^*(\lambda^2_G)-2\tau^*(\lambda^2_G))} \left( \pi^*(\lambda^2_G) e^{-\gamma(\tau^*(\lambda^2_G)-2\tau^*(\lambda^2_G))} \frac{(\lambda^1_G + \lambda^2_G) R - c}{r} \right).$$

Again, for sufficiently high $\lambda^2_G$, this bound is negative and exiting is dominant for firm 2. Given this, firm 1 finds it dominant not to exit at all $t \in [(n-1)\tau^*(\lambda^2_G), (n-1)\tau^*(\lambda^2_G) + \tau^*(\lambda^1_G))$, irrespective of its private history.

For the case of inconclusive news, we can again apply the approximation argument we used in the proof of Theorem 1 to show that $(\sigma_0, \sigma^*(\lambda^2_G))$ is the unique outcome that survives iterated deletion of conditionally dominated strategies. In this case, at time $2\tau^*(\lambda^2_G)$, the continuation payoff of firm 2 is bounded above by (omitting the dependence of $\pi^*$ and $\tau^*$ on $\lambda_B$)

$$\int_{2\tau^*(\lambda^2_G)}^{\tau^*(\lambda^2_G)} e^{-r(t-2\tau^*(\lambda^2_G))} \left( \left( \pi^*(\lambda^2_G) e^{-\gamma(t-2\tau^*(\lambda^2_G))} \lambda_G^2 + (1 - \pi^*(\lambda^2_G)) \lambda_B^2 \right) R - c \right) dt$$

$$+ e^{-r(\tau^*(\lambda^2_G)-2\tau^*(\lambda^2_G))} \left( \pi^*(\lambda^2_G) e^{-\gamma(\tau^*(\lambda^2_G)-2\tau^*(\lambda^2_G))} \frac{(\lambda^1_G + \lambda^2_G) R - c}{r} \right).$$
Recall that this is the continuation payoff at time $\tau^*(\lambda^2_G)$ in the hypothetical scenario in which firm 1 exits with probability one at time $\tau^*(\lambda^1_B)$; at that time, firm 2 perfectly learns the state and exits with no delay if $\omega_{\tau^*(\lambda^1_B)} = B$, because by assumption, $(\lambda^1_B + \lambda^2_B)R < 0$. Again, the integrand is negative, and by Lemma 3, the second term converges to zero as $\lambda^2_G \to \infty$. As a result, exiting is dominant for firm 2 at any time before $2\tau^*(\lambda^2_G)$ as soon as its beliefs fall short of $\pi^*(\lambda^2_G)$. Then, by the approximation argument, for sufficiently high $\lambda^2_G$, the belief of firm 1 is bounded away from $\pi^*(\lambda^1_B)$ at any time in $[\tau^*(\lambda^1_B), \tau^*(\lambda^1_G) + \tau^*(\lambda^2_G))$. Hence, it is dominant for firm 1 not to exit before $\tau^*(\lambda^1_G) + \tau^*(\lambda^2_G)$. The remainder of the proof follows from the same recursive argument as in the previous part.

If being a monopolist is profitable in both states, that is, $(\lambda^1_B + \lambda^2_B)R - c > 0$, then the relevant bound becomes

$$\int_{2\tau^*(\lambda^2_G)}^{\tau^*(\lambda^1_B)} e^{-r(t-2\tau^*(\lambda^2_G))}\left((\pi^*(\lambda^2_G)e^{-\gamma(t-2\tau^*(\lambda^2_G))})\lambda^2_G + \left(1 - \pi^*(\lambda^2_G)e^{-\gamma(t-2\tau^*(\lambda^2_G))}\right)\lambda^2_B \right) dt$$

$$+ e^{-r(\tau^*(\lambda^1_B)-2\tau^*(\lambda^2_G))}\left(\pi^*(\lambda^2_G)e^{-\gamma(\tau^*(\lambda^1_B)-2\tau^*(\lambda^2_G))}\left(\frac{\lambda^1_B + \lambda^2_G}{\lambda^1_B}\right)R - c\right)$$

$$+ e^{-r(\tau^*(\lambda^1_B)-2\tau^*(\lambda^2_G))}\left(1 - e^{-\gamma(\tau^*(\lambda^1_B)-2\tau^*(\lambda^2_G))}\pi^*(\lambda^2_G)\right)\left(\frac{2\lambda^1_B R - c}{r}\right).$$

Notice that as $\lambda^2_G \to \infty$, $\tau^*(\lambda^2_G) \to 0$, and $\pi^*(\lambda^2_G)\lambda^2_G \to 0$. Hence, in the limit, as $\lambda^2_G \to \infty$, the bound converges to

$$\frac{1 - e^{-r\tau^*(\lambda^1_B)}}{r}(\lambda^2_G R - c) + \frac{e^{-r\tau^*(\lambda^1_B)}}{r}(2\lambda_B R - c).$$

From (5) (see also Lemma 9 below),

$$\tau^*(\lambda^1_G) \geq \frac{1}{\gamma^1 + \lambda^1_G - \lambda_B} \ln \left(\frac{((\lambda^1_B - \lambda_B)((\lambda^1_G + \gamma)R - c))}{\gamma(c - \lambda_B R)}\right).$$

Replacing $\tau^*(\lambda^1_G)$ with this bound in (9), we obtain

$$- \frac{c - \lambda_B R}{r} + \frac{\lambda_B R}{r} \left(\frac{((\lambda^1_B - \lambda_B)((\lambda^1_G + \gamma)R - c))}{\gamma(c - \lambda_B R)}\right)^{-\frac{r}{\gamma + \lambda^1_G - \lambda_B}}.$$

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We now claim that there exists a set of \( \lambda_G \) such that this bound is negative. Assume that \( r > \gamma + \lambda_B \). Then, if \( \lambda_G^1 = 2\lambda_B \), (10) can be shown to be strictly negative. By continuity, we can then conclude that there exists a set of pairs \( \mathcal{L} \in (c/R, \infty) \) \( \times \) \( (c/R, \infty) \) \( , \lambda_G^2 > \lambda_G^1 \), for which \( (\sigma_1^0, \sigma_2^\pi(\lambda_G^2)) \) is the unique equilibrium.

**Proof of Theorem 2.B.** First, for any \( c \) and \( R \), we can choose \( \lambda_G^2 \) arbitrarily close to \( c/R \) such that \( 2\tau^*(\lambda_G^1) < \tau^*(\lambda_G^2) \).

Second, we show that for firm 1, exiting before \( \tau^*(\lambda_G^1) \) is a dominated action. As in Theorem 2.A, we argue that firm 1’s posterior along the history with no exit is bounded above \( \pi^*(\lambda_G^1) \) at any time before \( \tau^*(\lambda_G^1) \). The formal proof is in the Online Appendix. Here we provide an informal argument.

We show that in the limit as \( R/c \rightarrow \infty \) and \( \gamma \rightarrow \infty \), for \( \lambda_G^2 \) appropriately chosen, firm 2 not exiting always brings good news to firm 1, regardless of which strategy firm 1 expects firm 2 to play, among the strategy surviving our first round of deletion. Intuitively, the inference firm 1 draws from observing the action of firm 2 always concerns the state at some point in time in the past, and not about the prevailing state. Hence, as \( \gamma \rightarrow \infty \), this inference plays a limited role in determining firm 1’s posterior belief, which can be shown to be bounded away from \( \pi^*(\lambda_G^1) \) at any time before the first exit time \( \tau^*(\lambda_G^1) \).

Third, proceeding as in the proof of Theorem 2.A, we show that (8) is negative provided that \( R/c \) and \( r \) sufficiently high. That is,

\[
\int_{2\tau^*(\lambda_G^1)}^{\tau^*(\lambda_G^2)} e^{-r(t-2\tau^*(\lambda_G^1))} \left( \pi^*(\lambda_G^1) e^{-\gamma(t-2\tau^*(\lambda_G^1))} \lambda_G^2 R - c \right) \, dt \\
+ e^{-r(\tau^*(\lambda_G^1)-2\tau^*(\lambda_G^2))} \left( \pi^*(\lambda_G^2) e^{-\gamma(\tau^*(\lambda_G^2)-2\tau^*(\lambda_G^1))} \frac{(\lambda_G^1 + \lambda_G^2) R - c}{r} \right) < 0.
\]

Again, by the definition of \( \pi^*(\lambda_G^2) \), the first term is negative. By Lemma 9, for sufficiently high \( R/c \) and \( r > \lambda_G^1 \), the second term converges to zero whenever \( \lambda_G^2 \) is chosen to be arbitrarily close to \( c/R \). Crucially, the first integral in the equation above remains bounded away from zero as we take this limit because \( \tau^*(\lambda_G^1) \) is increasing in \( R/c \). Following the same steps as before, we can then prove that \( (\sigma_1^0, \sigma_2^\pi(\lambda_G^2)) \) is the unique strategy profile that survives iterated deletion of dominated strategies.

**Lemma 9.** If \( r > \lambda_G^1 \),

\[ e^{-(r+\gamma)\tau^*(\lambda_G^1)} \frac{\lambda_G^1 R}{r} \rightarrow 0, \]

as \( R \rightarrow \infty \) or \( c \rightarrow 0 \).
Proof. We are going to derive a lower bound for $\tau^*(\lambda^1_G)$ by identifying an upper bound for $\pi^*(\lambda^1_G)$. From (5), replacing $v(j(\pi^*(\lambda^1_G)))$ with 0, we obtain

$$\tau^*(\lambda^1_G) \geq \frac{1}{\gamma + \lambda^1_G} \ln \left( -\frac{\lambda^1_G}{r} + \frac{\lambda^1_G}{\gamma} \cdot \frac{\lambda^1_G R}{c} \right).$$

Hence,

$$e^{-(r+\gamma)\tau^*(\lambda^1_G)} \frac{\lambda^1_G R}{r} \leq \left( -\frac{\lambda^1_G}{r} + \frac{\lambda^1_G}{\gamma} \cdot \frac{\lambda^1_G R}{c} \right)^{-\frac{r+\gamma}{\gamma+\lambda^1_G} \frac{\lambda^1_G R}{r}}.$$

If $r > \lambda^1_G$, the right-hand side converges to 0 as $R \to \infty$ or $c \to 0$. In fact, the necessary condition for the right-hand side to converge to 0 is that as $R \to \infty$, $R/c$ converges to infinity in the order $O(R)$.

Proof of Proposition 2. We show that if (8) is negative for some $c^2$, then it is also negative for $c^{2'} > c^2$. Abusing notation, let $\pi^*(\lambda^2_G, c^2)$ and $\pi^*(\lambda^2_G, c^2)$ be the first exit time and the cutoff belief as a function also of the operating cost $c^2$. If

$$\int_{2\tau^*(\lambda^2_G, c^2)}^{\tau^*(\lambda^2_G, c^2)} e^{-r(t-2\tau^*(\lambda^2_G, c^2))} \left( \pi^*(\lambda^2_G, c^2) e^{-\gamma(t-2\tau^*(\lambda^2_G, c^2))} \lambda^2_G R - c^2 \right) dt$$

$$+ e^{-r(\tau^*(\lambda^2_G, c^2)-2\tau^*(\lambda^2_G, c^2))} \left( \pi^*(\lambda^2_G, c^2) e^{-\gamma(\tau^*(\lambda^2_G, c^2)-2\tau^*(\lambda^2_G, c^2))} \frac{\lambda^1_G + \lambda^2_G}{R} R - c^2 \right) < 0,$$

then exiting at $2\tau^*(\lambda^2_G, c^2)$ following a history with no customer is dominant for firm 2 also when its cost is $c^{2'} > c^2$. Hence, if firm 1 does not observe an exit at $2\tau^*(\lambda^2_G, c^2)$, it is dominant for it not to exit before $\tau^*(\lambda^1_G, c^2) + \tau^*(\lambda^2_G, c^1)$, regardless of the operating cost of firm 2. Applying the logic recursively, we can delete all strategies of firm 1 but $\sigma^1_0$. Then, any strategy of firm 2 other than $\sigma^2_{\pi^*(\lambda^2_G, c^2)}$ can be deleted. It follows that the only strategy profile that survives iterated deletion of dominated strategy is $(\sigma^1_0, \sigma^2_{\pi^*(\lambda^2_G, c^2)})$. For the case of inconclusive news, the result follows from combining the argument above and the approximation in the proof of Theorem 2.A.

Last, the proof of the comparative statics result with respect to $r^2$ follows from a similar argument.

A.2 Proof for Section 3

Proceeding in the same way as in the previous Section 4.1, we start by analyzing firm $i$’s best-reply problem to $\sigma^i_0$. In the continuation region, the value function of firm $i$,
when best replying to \( \sigma_0^i \) satisfies the Hamilton-Jacobi-Bellman equation
\[
rv^i(p) = (p(\lambda_G^i + \lambda_G^j) + (1 - p)2\lambda_B) (v^i(j(p)) - v^i(p)) - c \\
+ (p\lambda_G^i + (1 - p)\lambda_B) R \\
- (p(1 - p)(\lambda_G^i - \lambda_B + \lambda_G^j - \lambda_B) + p\gamma) v^i(p).
\]

As a result, whenever \( \lambda_G^i > c/R \), the optimal cutoff of firm \( i \) satisfies the following equation
\[
\hat{\pi}^*(\lambda_G^i, \lambda_G^j) = \frac{c - \lambda_B R}{(\lambda_G^i - \lambda_B + \lambda_G^j - \lambda_B) v^i(j(\hat{\pi}^*(\lambda_G^i, \lambda_G^j))) + (\lambda_G^i - \lambda_B) R} \\
- \frac{2\lambda_B v^i(j(\hat{\pi}^*(\lambda_G^i, \lambda_G^j)))}{(\lambda_G^i - \lambda_B + \lambda_G^j - \lambda_B) v^i(j(\hat{\pi}^*(\lambda_G^i, \lambda_G^j))) + (\lambda_G^i - \lambda_B) R}.
\]

Notice that in contrast to the case of unobservable customers, the optimal cutoff of firm \( i \) depends on both \( \lambda_G^i \) and \( \lambda_G^j \). In fact, using the implicit function, it is readily verified that \( \hat{\pi}^*(\lambda_G^i, \lambda_G^j) \) is decreasing both in \( \lambda_G^i \) and in \( \lambda_G^j \); and \( \lim_{\lambda_G^i \to \infty} \hat{\pi}^*(\lambda_G^i, \lambda_G^j) = \lim_{\lambda_G^j \to \infty} \hat{\pi}^*(\lambda_G^i, \lambda_G^j) = 0. \)

It can be shown that, again, \( \hat{\pi}^*(\lambda_G^i, \lambda_G^j) < 1 \) if and only if \( \lambda_G^i, R > c. \)

**Claim 1.** If \( \lambda_G^2 > \lambda_G^2 \), \( \hat{\pi}^*(\lambda_G^1, \lambda_G^1) > \hat{\pi}^*(\lambda_G^2, \lambda_G^2) \).

**Proof.** The value functions of the two firms are ranked pointwise, while the function \( j \) is identical for both of them. It follows that \( \hat{\pi}^*(\lambda_G^1, \lambda_G^1) > \hat{\pi}^*(\lambda_G^2, \lambda_G^2) \). \( \square \)

By the same argument as in **Lemma 2**, in any equilibrium, firm \( i \) never exits when its belief is strictly above \( \hat{\pi}^*(\lambda_G^i, \lambda_G^j) \). If \( \lambda_G^2 > \lambda_G^1 \), then \( \hat{\pi}^*(\lambda_G^1, \lambda_G^1) > \hat{\pi}^*(\lambda_G^2, \lambda_G^2) \), and by the same argument as in **Theorem 1** the strategy profile \( (\sigma_1^i(\hat{\pi}^*(\lambda_G^1, \lambda_G^1)), \sigma_0^i) \) is an equilibrium. It is easy to construct parametric examples for which \( \lambda_G^1 > \lambda_G^1 \), and \( (\sigma_0^1, \sigma_2^1(\hat{\pi}^*(\lambda_G^1, \lambda_G^1)) \) is not an equilibrium because at the belief \( \hat{\pi}^*(\lambda_G^1, \lambda_G^1) \), firm one prefers to exit rather than waiting to benefit from monopoly profits. The following lemma describes the construction of the mixed strategy equilibrium we discussed in Section 3.2.

**Lemma 10.** Suppose that \( \lambda_G^2 > \lambda_G^1 \) and \( \lambda_B = 0 \). If both \( (\sigma_0^1, \sigma_2^1(\hat{\pi}^*(\lambda_G^1, \lambda_G^1))) \) and \( (\sigma_1^1(\hat{\pi}^*(\lambda_G^1, \lambda_G^1)), \sigma_0^2) \) are equilibria, there exists a mixed strategy equilibrium such that

(i) firm 2 exits with probability \( q > 0 \) when the posterior belief is equal to \( \hat{\pi}^*(\lambda_G^1, \lambda_G^1) \);

(ii) both firms exit at a positive rate when the belief belongs to \( (\pi^1(\lambda_G^1 + \lambda_G^1), \hat{\pi}^*(\lambda_G^1, \lambda_G^1)) \), where \( \pi^1(\lambda_G^2 + \lambda_G^1) > 0 \) is the belief at which a monopolist optimally exits. 

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Proof. Recall that \( v^i : [0, 1] \to \mathbb{R} \) denotes the payoff associated with firm \( i \)'s best-reply to \( \sigma^G_0 \). Let \( W : [0, 1] \to \mathbb{R} \) denote the payoff associated with the monopolist problem.

The equilibrium we construct yields ex-ante expected payoffs equal to \((v^1(1), v^2(1))\). The probability \( q \) is chosen so that at the belief of \( \hat{\pi}^*(\lambda^1_G, \lambda^2_G) \), firm 1 is indifferent between exiting and waiting to exit as soon as the belief falls short of \( \hat{\pi}^*(\lambda^2_G, \lambda^1_G) \), provided that firm 2 does not exit then. That is, \( q \) satisfies

\[
\begin{align*}
&\int_0^{\gamma + \lambda^2_G - \lambda_B + \lambda^2_G - \lambda_B} \hat{\pi}^*(\lambda^1_G, \lambda^2_G) e^{-\gamma t} (\lambda^1_G + \lambda^2_G) \left( \frac{\lambda^1_G}{\lambda^1_G + \lambda^2_G} R + v^i(1) - \frac{1 - e^{-rt}}{r} \right) dt \\
&+ \left( 1 - \frac{\lambda^1_G + \lambda^2_G}{\lambda^1_G + \lambda^2_G + \gamma} \hat{\pi}^*(\lambda^1_G, \lambda^2_G) \left( 1 - e^{-(\lambda^1_G + \lambda^2_G + \gamma)} \ln \left( \frac{\ln(\hat{\pi}(\lambda^2_G, \lambda^2_G))}{\ln(\hat{\pi}(\lambda^2_G, \lambda^2_G))} \right) \right) \right) \\
&\left( -\frac{c}{r} \left( 1 - e^{-r \gamma + \lambda^2_G - \lambda_B + \lambda^2_G - \lambda_B} \right) \right) + e^{-r \gamma + \lambda^2_G - \lambda_B + \lambda^2_G - \lambda_B} q W(\hat{\pi}^*(\lambda^2_G, \lambda^1_G)) = 0.
\end{align*}
\]

Clearly, exiting at some belief in \((\hat{\pi}^*(\lambda^2_G, \lambda^1_G), \hat{\pi}^*(\lambda^1_G, \lambda^2_G))\) is suboptimal. At any belief \( p \in (\pi^1(\lambda^2_G + \lambda^1_G), \hat{\pi}^*(\lambda^2_G, \lambda^1_G)) \), firm \( i \) exits at a rate

\[
-\frac{p(\lambda^1_G + \lambda^2_G) \left( \frac{\lambda^1_G}{\lambda^1_G + \lambda^2_G} R + v^i(1) \right) - c}{W(p)}.
\]

It can be verified that this rate is positive and bounded for any \( p \in (\pi^1(\lambda^2_G + \lambda^1_G), \hat{\pi}^*(\lambda^2_G, \lambda^1_G)) \). Consequently, along a path with no costumers, there is a strictly positive probability that both firms are still in the market by the time the belief reaches any given \( p \in (\pi^1(\lambda^2_G + \lambda^1_G), \hat{\pi}^*(\lambda^2_G, \lambda^1_G)) \). However, because the rate diverges to infinity at \( \pi^1(\lambda^2_G + \lambda^1_G) \), as the denominator converges to zero, no firm will remain in the market at a belief lower than the monopoly cutoff. The rate is chosen to guarantee that each firm is indifferent between exiting and remaining in the market and exit in the next instant if no customer arrives. \( \Box \)